

Sloshing with marine and land-based applications

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ABSTRACT: The broad variety of sloshing in marine and land-based applications is reviewed. Theoretical and experimental aspects are considered. Estimation of eigenfrequencies and modes is discussed. A shallow-liquid approximation is used to study two-phase liquid flow in an oil-gas separator. A variational formulation is used to get a simple-form analytical approximation of the lowest natural sloshing mode in a 2D circular tank. The result is applied in a simplified linear modal model for the hydrodynamic tank force on a tanker vehicle during lane change. The effects of swash bulkheads and screens in Tuned Liquid Dampers are considered. A nonlinear analytically-based multimodal method is used to explain how sloshing depends on the liquid depth in 2D flow. The importance of 3D flow for square-base, nearly-square base and circular-base tanks is emphasized. For instance, swirling and chaos may occur. The scaling of slamming load effects from model to full scale in prismatic ship tanks is discussed.

1 INTRODUCTION

Sloshing must be considered for almost any moving vehicle or structure containing a liquid with a free surface and can be the result of resonant excitation of the tank liquid. Excitation with frequencies in the vicinity of the lowest natural frequencies is of primary practical interest. Resonant steady-state free surface flows in tanks in aircrafts, missiles and rockets have been the focus of extensive research. For these vehicles, sloshing will have a strong influence on the dynamic stability. Sloshing can also be the result of transient motion, for instance, when we spill coffee from our coffee cup. The fact that sloshing may strongly interact with the dynamics of the carrying “body” is evident when we carry a bucket of water. If sloshing starts in the bucket, it is difficult for us to stop it.

Sloshing in separators on-board floating oil and gas production platforms affects the efficiency of oil-gas separators. Structures fixed onshore may be exposed to sloshing if an earthquake occurs. Sloshing effects are included in design loads for LNG storage tanks. Large scale sloshing in a lake with steep sides may be the result of a landslide or earthquake. Under dam construction such circumstances should be factors to investigate. Large scale sloshing may also occur in harbors and lakes and even on an ocean scale. Sloshing in a container can be used to damp out wind-induced motions of tall buildings. A tuned liquid damper, TLD, is a well known concept in the civil engineering world. Its physical basis is similar as for an anti-rolling tank on board a ship.

A partially filled ship tank can experience very violent liquid motion when the lateral and angular ship motions contain sufficient energy in the vicinity of the highest natural period for the liquid motion inside the tank. A consequence is wave breaking, spray and mixing of air (or gas) and liquid. Actually, extreme cases with bubbles everywhere in the liquid have been experimentally observed. Impact between the liquid and the tank structure is important, for instance, in the design of prismatic LNG tanks.

For both resonant and non-resonant sloshing excitations, the tank shape, the level of filling and the characteristics of the tank motion, e.g. amplitude and frequency content, make up the

principal parameters that determine the nature of the free-surface flow. The relative importance of the different parameters depends on the characteristics of the flow, i.e. the response. There is a clear difference between sloshing in shallow-liquid and higher filling level conditions. For small ratios between liquid depth and tank length and an excitation frequency around resonance, a hydraulic jump or bore, which travels back and forth in the tank, is formed when the excitation amplitude is beyond a threshold value. When the steep front of the bore hits the tank wall, an impact occurs and a thin vertical jet shoots upwards. When the liquid depth is finite and the liquid motion is two-dimensional, the free surface motion resembles a standing wave. However, nonlinearities prevent nodal points. Swirling or rotary waves is a special feature of three-dimensional flow, for instance, in a spherical, vertical circular, square-base or nearly square-base tank.

When the interior tank surface is smooth and there are no internal structures as stringers obstructing the liquid flow and wave breaking does not occur, the viscous damping of the liquid motions is small. The damping for a smooth tank can be very small at finite depth, i.e. it takes a very long time for transience to die. The damping increases in general with decreasing liquid depth.

Because sloshing is a typical resonance phenomenon, it is not necessarily the most extreme ship motions or external wave loads that cause the most severe sloshing. This implies that external wave-induced loads can in many practical cases be described by linear theory. The ship motions excite sloshing, which in return affects the ship motions. Ships equipped with anti-rolling tanks utilize this effect. The sloshing-induced roll moment on the vessel will cause roll damping by properly choosing the highest natural sloshing period close to the roll natural period. FPSO units sometimes have several partially filled tanks during operation. The wave-induced motions and loads on these ships will then be influenced by the liquid dynamics in the tanks. Because ship motions can strongly affect the mean and slowly-varying wave drift forces and moments, sloshing may also matter in a station-keeping analysis.

Ingress of water to a damaged ship and resulting dynamics of the water on large deck areas can result in sloshing and affect the capsizing process. Resonances in moonpools and in the gap between a ship and a terminal are examples on external sloshing for marine structures.

The hydrodynamics of sloshing is complicated. Its understanding requires a combination of theory, Computational Fluid Dynamics (CFD) and experiments. Tank design is often based on experiments. However, scaling of impact slosh pressures from model to full scale is not completely understood. Combination of hydrodynamics and structural mechanics is an important aspect of tank design.

2 LINEAR THEORY

Even for linear potential flow theory of an incompressible liquid, there exist few analytical solutions of the liquid sloshing boundary value problem. Examples are for 2D and 3D rectangular tanks, vertical circular cylindrical tanks, annular and sectored upright circular tanks, wedge cross-sections with semi-apex angles 45° and 60° .

The flow in a U-tube can be described analytically by assuming one-dimensional flow. The U-tube designs by the Rolls-Royce Company INTERING use active control when the ship rolls with a period larger than the uncoupled natural period for the liquid motion in the tank. Valves at the two reservoirs are alternatively opened and closed. When the valves are closed on either the port or starboard side, it nearly latches the water in the U-tube. This effect can be theoretically described by accounting for the compressibility in the closed air chamber (Faltinsen & Timokha 2009).

2.1 Oil-gas separators

Considering a shallow-liquid approximation opens new possibilities for deriving analytical solutions, e.g. estimating the natural sloshing frequencies and modes. An example is an oil-gas separator involving longitudinal two-phase liquid flow in a horizontal circular cylinder with constant cross-section, internal radius R_0 and length L . The upper and lower liquid layers are oil and water, respectively. We denote the filling heights of the two liquids as h_i , $i = 1, 2$ with sub-

scripts 1 and 2 for the upper and lower liquids, respectively. Figure 1 (solid line) presents numerical data for the two natural periods associated with the longest natural standing wave as a function of h_2 . Calculations are made with $R_0 = 2.0m$, $L = 14.0m$, $\rho_1 = 830kgm^{-3}$, $\rho_2 = 1000kgm^{-3}$, $h_1 + h_2 = R_0$ (half-filled tank). When we assume either that the two liquids have the same density or there are no free-surface wave effects, there is only a single natural period for this natural mode. The corresponding results are presented by a dotted line in parts a and b of this figure, respectively. We can see that one of the natural periods becomes very large and resonant oscillations may be excited due to slow-drift motions of the platform where the oil-gas separator is installed. This period is close to what we can predict by neglecting the free-surface wave effect (see, part b). The other natural period (part a) with two liquid phases will be excited by linear wave-frequency motion of the platform. We note that the lower natural period deviates little from the calculated natural period without accounting for two liquid phases. The fact that wave-induced platform motions may cause resonance oscillations inside an oil-gas separator and results in operational limitations means that damping devices are normally introduced inside the separator. An example is perforated vertical plates placed at different positions along the separator. The damping is associated with flow separation and is further discussed in section 3.

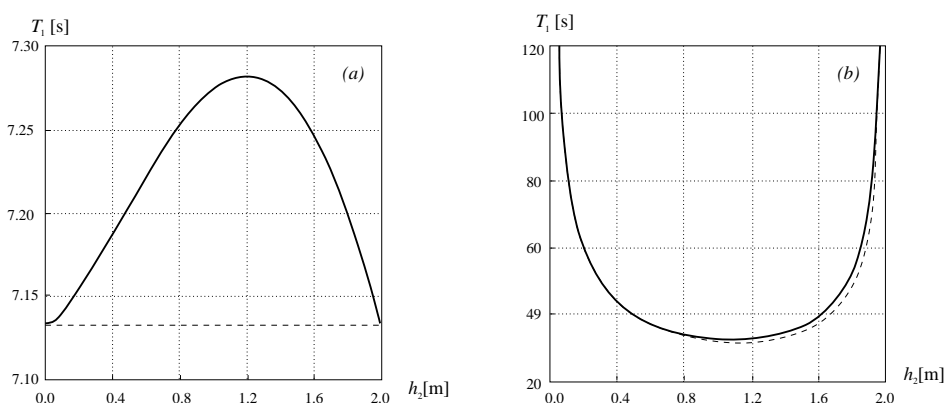


Figure 1. Calculated natural periods of a gas-oil separator for the longest longitudinal standing wave in a horizontal circular cylindrical half-filled tank. A shallow-liquid approximation is used. The results are presented as a function of the filling height of water. Solid lines are for two-phase flow. The dotted line on the left figure is based on neglecting the interface between the liquids and considering one liquid. The dotted line in the right figure is based on neglecting the free-surface waves (Faltinsen & Timokha 2009).

2.2 Domain decomposition, analytical continuation and interior structures

If the tank domain can be divided into sub-domains in which an analytical solution form is possible, analytical approximate natural sloshing modes can be constructed by the domain decomposition technique. Solutions in the different sub-domains are combined by requiring that both the velocity potential and the normal velocity are continuous at the interface between the sub-domains. The procedure is, for instance, relevant for a swimming pool on a cruise vessel with a wading region along the rim of the pool.

How can we in a simple as possible way estimate the natural frequencies when no analytical solution exists? First of all, we can use benchmark numerical results. Another possibility for some tank shapes consists of using analytical continuation for geometrically close tank shapes. This procedure can be used to estimate the effect of chamfers, tapering and inclination of the tank bottom. Analytical continuation cannot be used to estimate the effect of interior structures such as baffles, pump towers and screens on the natural frequencies. However, if the interior structure does not have a net source/sink effect on the flow and the cross-dimensions of the interior structure are small relative to both the main tank dimensions and the wave length of the considered sloshing mode, the natural frequencies can be expressed in terms of the added mass coefficients and displaced mass of the interior structure. A similar expression is possible for slender structures with small transverse cross-sections. In order to derive the corresponding

formulae, we must know the natural frequencies and modes for the tank without the interior structure.

2.3 Variational formulation and rollover of tanker vehicles

A possibility to get the upper and lower bounds of natural frequencies is to use variational formulations of the corresponding spectral problem. We should then substitute analytical natural modes found for another tank shape into the variational formulations. However, the difference between these upper and lower bounds may be too large to provide a satisfactory estimate. If so, as an alternative, we can also apply the variational formulations directly by using special test functions φ_* that only need to satisfy liquid volume conservation, i.e. the test functions are not the analytical natural sloshing modes. The lowest natural frequency σ_1 can then be estimated by minimizing the functional

$$\sqrt{g \int_{Q'_0} (\nabla \varphi_*)^2 dQ / \int_{\Sigma_0} \varphi_*^2 dx dy}, \quad (1)$$

where g is the acceleration of gravity, Q'_0 is the mean liquid volume and Σ_0 is the mean free surface. If a test function φ_* is identical to the eigenfunction for the lowest mode, the mathematical expression for σ_1 is exactly the same that follows from energy conservation.

We will illustrate this technique for sloshing in 2D circular tanks. The used special test functions are assumed to be horizontal dipoles in infinite fluid with singularities in the tank's centre-plane at a distance a above the mean free surface. This is the a -parametric family of test functions. The parameter a is generally unknown and should be determined by minimizing the functional (1). Table 1 shows very good agreement with the numerical benchmark results by McIver (1989). How to choose good test functions in the general case is not always obvious. The test functions in the above-mentioned case were constructed based on experimental observations of liquid path lines.

Table 1 Non-dimensional lowest eigenvalue $\kappa_1 R_0 = \sigma_1^2 R_0 / g$ for two-dimensional sloshing in a circular tank as a function of the ratio between maximum liquid depth h and radius R_0 (McIver 1989). Estimate of $\kappa_1 R_0$ is based on minimizing the functional (1) by a . The approximate value of the lowest frequency is also given together with the vertical position a of the dipole singularity above mean free surface (Faltinsen & Timokha 2009).

h / R_0	$\kappa_1 R_0$ (McIver, 1989)	Variational method (Faltinsen & Timokha 2009)	
		$\kappa_1 R_0$	a / R_0
0.2	1.04385	1.044012	1.836
0.4	1.09698	1.097780	1.673
0.6	1.16268	1.164845	1.511
0.8	1.24606	1.250770	1.348
1.0	1.35573	1.364880	1.185
1.2	1.50751	1.524338	1.018
1.4	1.73463	1.765255	0.842
1.6	2.12374	2.182000	0.651
1.8	3.02140	3.153600	0.427

The dipole-type approximate natural mode can be further used for simulating a lane change of a tanker vehicle with a partially filled tank with a circular cross-section. A single-mode approximation of the wave elevation is then expressed as $\zeta(y, t) = \beta_1(t) [R_0 y / (y^2 + a^2)]$ where y is a horizontal coordinate in the mean free surface with $y = 0$ in the middle of the free surface. The use of the linear modal theory (Faltinsen & Timokha 2009) gives that the generalized free-surface coordinate $\beta_1(t)$ satisfies the differential (modal) equation $\ddot{\beta}_1(t) + \sigma_1^2 \beta_1(t) = -P_1^0 \ddot{\eta}_2(t)$, where $\ddot{\eta}_2(t)$ is the given sway acceleration of the tank and the hydrodynamic coefficient P_1^0 can be expressed analytically. The horizontal hydrodynamic force is $F_2 = -m_l \ddot{\eta}_2(t) - \lambda_{21} \beta_1(t)$, where m_l is the liquid mass and λ_{21} is an analytically determined hydrodynamic coefficient. The results for the maximum horizontal hydrodynamic force due to a maneuver involving a lane change (see, the graphs of $\eta_2(t)$ and $\ddot{\eta}_2(t)$) are shown

