Numerical and experimental uncertainty analysis for the prediction of resistance and wave profile of a surface ship model

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Abstract: The numerical accuracy and experimental uncertainty are increasingly paid attention for both CFD and EFD. In recent years, the International Towing Tank Conference (ITTC) issued some recommended procedures for CFD and EFD to promote the prediction of ship hydrodynamics more credible.

In this paper, we shall present some of our recent study in the CFD and EFD uncertainty analysis on the prediction of resistance and wave profile of a benchmark surface ship model (DTMB5415). The unsteady viscous flow around the hull with free surface is studied through solving Reynolds Averaged Navier-Stokes (RANS) equations numerically. The uncertainty analysis of CFD computation is mainly focused on the grids. Structurized H-O type grids are used in the simulation. The grids are separately refined in 3 directions to demonstrate respective influence. Three solutions are used in each direction for the uncertainty analysis. The Generalized Richardson Extrapolation method is adopted to estimate the errors. The integrated uncertainty of CFD is presented. Meanwhile, the tests on resistance and wave profile of the model are carried out in CSSRC towing tank. The test definition and data uncertainty analysis are performed according to the technical procedure of ITTC. For the benchmark comparison, the results of test are also compared with other published results in terms of uncertainty.

Keywords: CFD; model test; uncertainty analysis; surface ship; resistance; wave profile

1. Introduction

Model test is still the most used and widely accepted approach in ship hydrodynamics investigations, but there is no doubt that the role of Computational Fluid Dynamics (CFD) in design problems is growing. In Experimental Fluid Dynamics (EFD) it is already a standard practice to indicate the uncertainty of a specific measurement. Therefore, it is hard to believe that CFD may establish itself as a reliable alternative and complement to model testing without indicating the numerical uncertainty of a given prediction.

It is commonly accepted that the numerical uncertainty of a CFD prediction has three components: the round-off error, the iterative error and the discretization error. The first two error sources are often neglected in present CFD for ship hydrodynamics. The discretization error is a consequence of the approximations made to transform the partial differential equations of the continuum formulations into a system of algebraic equations, and it is determined mainly by the grids used for simulation. In practical applications of complex turbulent flows, it is the most important of the three components. Therefore, the main attention has been lately focused on the estimation of the discretization error.

In recent years, the International Towing Tank Conference (ITTC) issued some recommended procedures to promote the prediction of ship hydrodynamics more credible. For EFD uncertainty analysis, the procedures are used widely and getting mature. Later, the, 23rd ITTC Resistance Committee presented an interim procedure with guide illustration for CFD uncertainty analysis on the prediction of resistance and flow of a ship model[1,2]. The procedures are largely based on the work of Stern and co-workers[3,4,5]. It addresses the issue of computation verification for a particular problem primarily based on grid and iterative convergence. This procedure was first recommended to the Gothenburg 2000 Workshop (G2K) where 7 of the 20 participants attempted to follow it[6,7].

Since then, several works have been done for CFD uncertainty analysis based on ITTC recommended procedures. Simonsen and Stern performed verification and validation for RANS

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maneuvering simulation of Esso Osaka hull[8]. Three solutions were used for analysis with about 2,000,000 cells for the finest grid. The work was mainly focused on the process of verification and a. double body approximation was adopted for the free surface. The uncertainty procedure was demonstrated for the DARPA SUBOFF submarine configuration by Van et al[9]. who performed calculations for both the axisymmetric bare hull configuration and the configuration with four stern appendages, but only the uncertainty analysis for the bare hull Cp was conducted. For applying the ITTC procedure, Zhu et al., Zhang et al. newly performed CFD uncertainty analysis for resistance and flow field of SUBOFF body[10,11].

The procedure recommended by the ITTC is not without controversy. One of the biggest drawbacks of the method is that it is formulated for regular structured grids and relies on Richardson extrapolation for quantifying the errors. There is a fundamental requirement that the grids used to estimate the uncertainty be of a similar nature. Three grids, all within the asymptotic range, are needed to determine the order of accuracy of the calculation. A constant grid refinement ration is also needed implicitly in the derivation. Such requirements are difficult to achieve for complicated problems. This is not to say that uncertainty estimates for CFD are not important. As addressed in recent ITTC resistance committee report[12], the current recommended procedure is just an interim basis and the study for CFD uncertainty estimation needs to be encouraged.

In present paper, some of our recent study in the CFD and EFD uncertainty analysis on the prediction of resistance and wave profile of a benchmark surface ship model (DTMB5415) is presented. The uncertainty analysis of CFD computation is mainly focused on the grids. Structurized H-O type grids are used in the simulation. It is a little bit different from the ITTC recommended procedure that the grids are separately refined in 3 directions to demonstrate respective influence. Three solutions are used in each direction for the uncertainty analysis. The Generalized Richardson Extrapolation method is adopted to estimate the errors. The integrated uncertainty of CFD is presented. Meanwhile, the tests on resistance and wave profile of the model are carried out in CSSRC towing tank. The test definition and data uncertainty analysis are performed according to the technical procedure of ITTC. For the benchmark comparison, the results of test are also compared with other published results in terms of uncertainty.

2. Verification and Validation Methodology

The verification and validation methodology and procedure are quoted from ITTC Quality Manual. The definition of the variables here is referred to the reference[1].The simulation error δ_S is defined as the difference between a simulation result S and the true T and is composed of modeling error δ_{SM}, and numerical errors δ_{SN}

$$\delta_S = S - T = \delta_{SM} + \delta_{SN} \tag{1}$$

For certain conditions, both the sign and magnitude of the numerical error can be estimated as,

$$\delta_{SN} = \delta_{SN}^* + \varepsilon_{SN} \tag{2}$$

where δ_{SN}^* is an estimation of δ_{SN} and ε_{SN} is the error in that estimation. The benchmark value of the numerical simulation S_C is gotten by the following correction

$$S_C = S - \delta_{SN}^* \tag{3}$$

Verification is to assess the numerical uncertainty U_{SN} in the simulation. The δ_{SN}^* and its uncertainty U_{S_CN} are also to be estimated. For the uncorrected simulation approach, numerical error is decomposed into the contributions from iteration number δ_I, grid size δ_G, time step δ_T, and other parameters δ_P. The numerical uncertainty in the simulation can be written as the following expression.

$$U_{SN}{}^2 = U_I{}^2 + U_G{}^2 + U_T{}^2 + U_P{}^2 \tag{4}$$

For the corrected simulation approach, the benchmark value of the numerical simulation S_C is gotten by the result correction. The δ^*_{SN} and $U_{S_C N}$ are given by

$$\delta^*_{SN} = \delta^*_I + \delta^*_G + \delta^*_T + \delta^*_P \tag{5}$$

$$U_{S_C N}{}^2 = U_{I_C}{}^2 + U_{G_C}{}^2 + U_{T_C}{}^2 + U_{P_C}{}^2 \tag{6}$$

Validation is to assess the modeling uncertainty U_{SM} in the simulation by using benchmark experimental data. The modeling error δ_{SM} is also to be estimated. The comparison error E is defined as the difference between the experimental data D and simulation result S.

$$E = D - S = \delta_D - (\delta_{SM} + \delta_{SN}) \tag{7}$$

The validation is estimated by comparing E with the validation uncertainty U_V, which is given by

$$U_V{}^2 = U_D{}^2 + U_{SN}{}^2 \tag{8}$$

If $|E| < U_V$, the validation is achieved at the U_V level. If $U_V << |E|$, the $E \approx \delta_{SM}$ can be taken to improve the modeling. For the corrected simulation, the equations (7) and (8) are modified as following.

$$E_C = D - S_C = \delta_D - (\delta_{SM} + \varepsilon_{SN}) \tag{9}$$

$$U_{V_C}{}^2 = U_{E_C}{}^2 - U_{SM}{}^2 = U_D{}^2 + U_{S_C N}{}^2 \tag{10}$$

3. CFD Uncertainty Analysis

3.1 Code, Geometry, Conditions, and Benchmark Data

The computation is realized in the FLUENT, a general-purpose CFD package that has been widely adopted in the simulation of ship flow. The unsteady RANS solver is based on a cell-centered finite volume discretization, with the second-order upwind difference scheme for the convection term and the centric difference scheme for the dissipation term. Multigrid acceleration algorithm is used to solve the difference equation system. The couple of pressure and velocity was solved by *SIMPLE* algorithm. The turbulence is modeled through Renormalization Group (RNG) $k-\varepsilon$ two-equation turbulence model in conjunction with the wall function approach for the near-wall simulation. The VOF method is used for the free surface treatment.

The geometry is the naval combatant model DTMB5415. This flow was one of the test cases in the Gothenburg 2000 workshop on CFD in Ship Hydrodynamics. The conditions for the computations are Froude number $Fr=0.41$, Reynolds number $Re=15.381*10^6$, the sinkage and trim of the model test are taken into consideration in the calculations of resistances. The variables selected for verification and validation are resistance R_T (integral variable) and wave profile ζ (point variable).

Meanwhile, the tests on resistance and wave profile of the model are carried out in CSSRC towing tank. The test is a part of the plan of ITTC Worldwide Series for Identifying Facility Biases organized by Resistance Committee of 24th ITTC. The test definition and data uncertainty analysis are performed according to the technical procedure of ITTC.

3.2 Computational Grids

In most of published works on ship CFD uncertainty analysis, the computational grids are assumed to be homogenous, i.e., the effect of the grid refinement in each direction is taken for the same.

The present grid is a body-fitted, multi-block and structurized H-O topology system. The grids are separately refined in three directions to demonstrate respective influence. In each direction three grids are used for uncertainty analysis. The sizes of grid 1 (finest), 2 (middle) and 3 (coarsest) are 71*32*45, 100*32*45 and 142*32*45 in the ξ-direction; 100*23*45, 100*32*45 and 100*45*45 in the ζ-direction; 100*32*32, 100*32*45 and 100*32*64 in the η-direction. The grid refinement ratio is taken as $r_\xi = r_\zeta = r_\eta = \sqrt{2}$. Clustering is used near the bow and stern in the ξ-direction, near the free surface in the ζ-direction, and at the hull in the η-direction. All the cell numbers mentioned in the ξ-direction are only for the range of ship model length.

3.3 Verification and Validation of Resistance Prediction

Here, verification of the model resistance R_T prediction is first performed by studying on the grid convergence in three directions, respectively. The results are summarized in the table 1.

Table 1 Grid convergence study on the total resistance R_T of DTMB5415 model

	ξ-direction		ζ-direction		η-direction	
Grid Series	Cell Number	R_T(N)	Cell Number	R_T(N)	Cell Number	R_T(N)
1	142	147.4	45	148.0	64	148.1
2	100	148.4	32	148.4	45	148.4
3	71	150.9	23	150.1	32	150.0

First, in the ξ-direction, the differences of total resistance between the neighboring solutions are

$$\varepsilon_{\xi 21} = R_{T\xi 2} - R_{T\xi 1} = 148.4 - 147.4 = 1.0 \tag{11}$$

$$\varepsilon_{\xi 32} = R_{T\xi 3} - R_{T\xi 2} = 150.9 - 148.4 = 2.5 \tag{12}$$

The convergence ratio in the ξ-direction can be calculated as

$$R_{\xi 1} = \varepsilon_{\xi 21} / \varepsilon_{\xi 32} = 0.400 \tag{13}$$

Because of $0 < R_{\xi 1} < 1$, the grids in the ξ-direction are monotonic convergence. The generalized Richardson extrapolation is used to estimate the error $\delta^*_{RE\xi 1}$ and the order of accuracy p_ξ,

$$\delta^*_{RE\xi 1} = \frac{\varepsilon_{\xi 21}}{r_\xi^{p_\xi} - 1} = \frac{1.0}{\sqrt{2}^{2.644} - 1} = 0.667 \tag{14}$$

$$p_\xi = \frac{\ln(\varepsilon_{\xi 32}/\varepsilon_{\xi 21})}{\ln(r_\xi)} = \frac{\ln(2.5/1.0)}{\ln\sqrt{2}} = 2.644 \tag{15}$$

The correction factor C_ξ is

$$C_\xi = \frac{r_\xi^{P_\xi} - 1}{r_\xi^{P_{\xi est}} - 1} = \frac{\sqrt{2}^{2.644} - 1}{\sqrt{2}^2 - 1} = 1.500 \tag{16}$$

where $p_{\xi est} = p_{th} = 2$.

For $C_\xi = 1.500$ considered as sufficiently greater than or less than 1 and lacking confidence, U_ξ is estimated and not δ_ξ

$$U_{\xi} = \left[\left|C_{\xi}\right| + \left|\left(1 - C_{\xi}\right)\right|\right]\left|\delta^{*}_{RE\xi 1}\right| = [1.500 + 0.500]*0.667 = 1.333 \tag{17}$$

U_{ξ} is 0.886% *D*.

Similarly, the verification in the ζ and η directions is undertaken. All the results of the verification of total resistance R_T are listed in the table 2. From the table we can see that in all the three directions, the level of verification is relatively small, less than 1%*D*.

Table 2 Verification of the total resistance R_T of DTMB5415 model

	R	*p*	*C*	δ_{RE}*	δ_{RE}* (%*D*)	*U*	*U*(%*D*)
ξ-direction	0.400	2.644	1.500	0.667	0.443	1.333	0.886
ζ-direction	0.235	4.175	3.250	0.123	0.082	0.676	0.450
η-direction	0.188	4.830	4.333	0.069	0.046	0.529	0.352

Validation is then performed by using the prediction of R_T. The benchmark data provided by CSSRC for the total resistance of DTMB5415 model at *Fr*=0.41 is *D*=150.4N with uncertainty U_D=1.866%*D*.

In the ξ-direction, the comparison error is calculated as

$$E_{\xi} = D - R_{T\xi} = 150.4 - 147.4 = 3.0 = 2.00\%D \tag{18}$$

The validation uncertainty is calculated as

$$U_{V\xi} = \sqrt{U_{\xi}^{\ 2} + U_{D}^{\ 2}} = \sqrt{1.333^2 + 2.806^2} = 3.107 = 2.07\%D \tag{19}$$

On account of $\left|E_{\xi}\right| < U_{V\xi}$, the simulation result in the ξ-direction is validated.

The same work can be done in the ζ and η direction. The validation results of total resistance R_T prediction are listed in the table 3.

Table 3 Validation of the total resistance prediction for DTMB5415 model

	E(%*D*)	U_V(%*D*)	U_D(%*D*)	*U*(%*D*)
ξ-direction	2.00	2.07	1.866	0.886
ζ-direction	1.60	1.92		0.450
η-direction	1.53	1.90		0.352

In all the three directions, the value *E* is always positive, i.e., the simulation makes an underestimation. Presumably the modeling errors such as resolution of the wave field and inclusion of effects of model free condition may be addressed to reduce the *E*.

3.4 Verification and Validation of Wave Profile Simulation

In this paper, the study on grid convergence for the wave profile is only carried out in the ξ-direction within the range of ship model length. The results of the study on grid convergence for the wave profile along *y*=0.172L_{PP} are shown in the figure 1. The data of benchmark model test is also depicted in the figure.

The differences of wave elevation between the neighboring solutions are

$$\varepsilon_{\xi 21} = Z_{\xi 2} - Z_{\xi 1} \tag{20}$$

$$\varepsilon_{\xi 32} = Z_{\xi 3} - Z_{\xi 2} \tag{21}$$

The L2 norm of the solution changes over the *N* points is

$$\left\| \varepsilon_{\xi} \right\|_2 = \left[\sum_{i=1}^{N} {\varepsilon_{\xi i}}^2 \right]^{1/2} \tag{22}$$

The convergence ratio can be calculated as

$$R_{\xi 1} = \frac{\left\| \varepsilon_{\xi 21} \right\|_2}{\left\| \varepsilon_{\xi 32} \right\|_2} = 0.513 \tag{23}$$

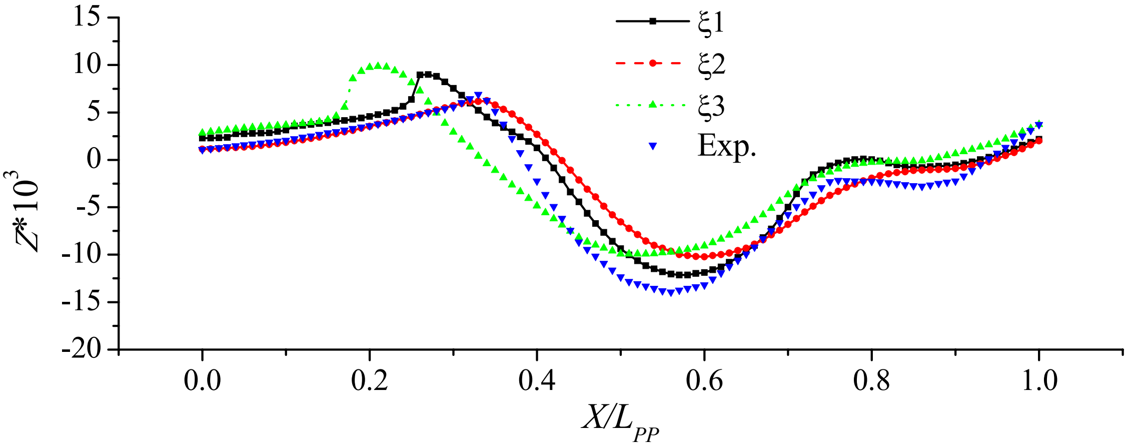

Figure 1 Grid convergence study for wave profile at y=0.172L_{PP}

Because of $R_{\xi 1} < 1$, the grids show convergence. The error and spatial order of accuracy for the wave profile are estimated from the L2 norm of solution changes

$$\delta^*_{RE\xi 1} = \frac{\left\| \varepsilon_{\xi 21} \right\|_2 / \sqrt{N}}{r_{\xi}^{\langle p_\xi \rangle} - 1} = \frac{1.818}{\sqrt{2}^{1.928} - 1} = 1.912 \tag{24}$$

$$\langle p_\xi \rangle = \frac{\ln\left(\left\| \varepsilon_{\xi 32} \right\|_2 / \left\| \varepsilon_{\xi 21} \right\|_2 \right)}{\ln(r_\xi)} = 1.928 \tag{25}$$

Correction factor is computed using order of accuracy p_ξ and $p_{\xi est} = 2.0$

$$\langle C_\xi \rangle = \frac{r_{\xi}^{\langle p_\xi \rangle} - 1}{r_{\xi}^{p_{\xi est}} - 1} = \frac{\sqrt{2}^{1.928} - 1}{\sqrt{2}^{2} - 1} = 0.951 \tag{26}$$

On account of $\left| 1 - \langle C_\xi \rangle \right| = 1 - 0.951 = 0.049 < 0.125$, the uncertainty is estimated with factor of safety,

$$U_\xi = F_S \left| \delta^*_{RE\xi 1} \right| = 1.25 * 1.912 = 2.390 \tag{27}$$

U_ξ is 17.15% $|Z|_{D\max}$.

The profile-averaged values from verification of wave profile along y=0.172L_{PP} for DTMB5415 model are listed in table 4. From the table we can see that in the ξ-direction, the level of verification is relatively large, more than 15% $|Z|_{D\max}$.

Table 4 Profile-averaged values from verification of the wave profile along y=0.172L_{PP}

$R_{\xi 1}$	p_{ξ}	C_{ξ}	$\delta_{RE\xi}^{*}$	$\delta_{RE\xi}^{*}$ (%$\|Z\|_{D\max}$)	U_{ξ}	U_{ξ}(%$\|Z\|_{D\max}$)
0.513	1.928	0.951	1.912	13.72	2.390	17.15

The profile-averaged comparison error is calculated as

$$E_{\xi} = \left\| Z_D - Z_{\xi} \right\|_2 \Big/ \sqrt{N} = 1.952 = 14.01\% |Z|_{D\max} \tag{28}$$

The profile-averaged validation uncertainty is calculated as

$$U_{V\xi} = \sqrt{U_{\xi}^{\,2} + U_{D}^{\,2}} = \sqrt{17.15^2 + 4.0^2} = 17.61\% |Z|_{D\max} \tag{29}$$

The uncertainty of the benchmark data for the wave profile is assumed to be U_D=4.0%$|Z|_{D\max}$.

The results for profile-averaged validation of wave profile along y=0.172L_{PP} are listed in the table 5. On account of $|E|_{\xi}<U_{V\xi}$, the simulation profile-averaged results of wave profile along y=0.172L_{PP} in the ξ-direction are validated. It can be seen that U_{ξ} is the main body of the total U_V, thereby the reduction in U_V would require the decrease of U_{ξ}.

Table 5 Profile-averaged validation of wave profile along y=0.172L_{PP}

E_{ξ}(%$\|Z\|_{D\max}$)	$U_{V\xi}$(%$\|Z\|_{D\max}$)	U_D(%$\|Z\|_{D\max}$)	U_{ξ}(%$\|Z\|_{D\max}$)
14.01	17.61	4.0	17.15

The point comparison error is compared to validation uncertainty in the figure 2. Most of points on the wave profile are validated. The largest errors occur at the crests and trough regions.

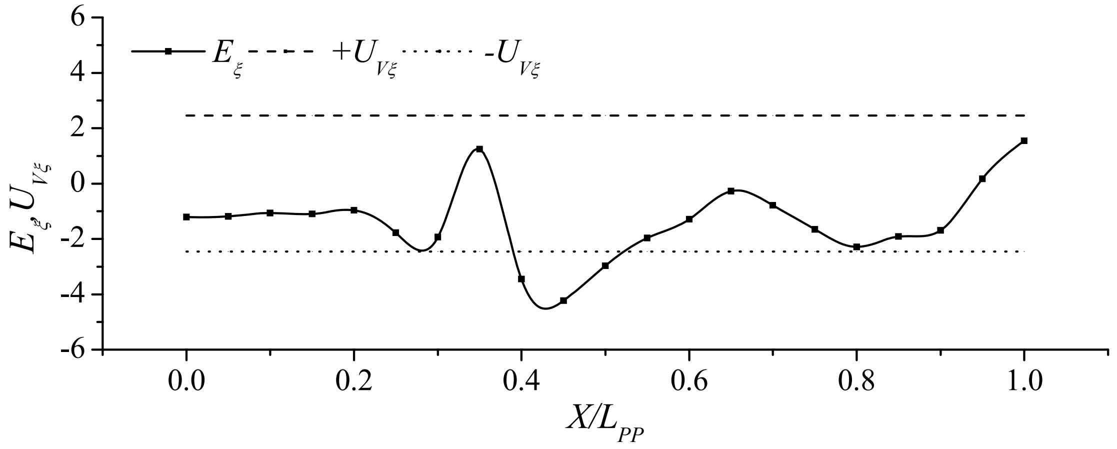

Figure 2 Validation of wave profile at y=0.172L_{PP}

4. Benchmark Model Tests

The tests on resistance, sinkage and trim, and wave profile of the model are carried out in CSSRC towing tank. The test is a part of the plan of ITTC Worldwide Series for Identifying Facility Biases organized by Resistance Committee of 24th ITTC. The test definition and data uncertainty analysis are performed according to the technical procedure of ITTC.

Uncertainty analysis of total resistance for DTMB5415 is performed at speeds of Fr=0.10, 0.28 and 0.41. The results are listed in table 6. The results of uncertainty analysis of total resistance for DTMB5512 model carried out by IIHR are also listed in the table. The comparison indicates that the level of uncertainty of CSSRC's data is quite similar to that of IIHR.

Table 6 Model test Uncertainty of total resistance C_T

Fr	C_T		U_{CT}		U_{CT}/C_T	
	DTMB5145 (CSSRC)	DTMB5512 (IIHR)	DTMB5145 (CSSRC)	DTMB5512 (IIHR)	DTMB5145 (CSSRC)	DTMB5512 (IIHR)
0.10	$4.217*10^{-3}$	$4.645*10^{-3}$	$1.600*10^{-4}$	$2.381*10^{-4}$	3.795%	5.12%
0.28	$4.207*10^{-3}$	$4.669*10^{-3}$	$7.266*10^{-5}$	$5.061*10^{-5}$	1.727%	1.08%
0.41	$6.443*10^{-3}$	$6.834*10^{-3}$	$1.202*10^{-4}$	$4.572*10^{-5}$	1.866%	0.67%

5. Concluding Remarks

Some of our recent studies in CFD and EFD uncertainty analysis on the prediction of resistance and wave profile of a benchmark surface ship model (DTMB5415) are presented in this paper. The uncertainty analysis of CFD computation is mainly focused on the grids. The grids are separately refined in three directions and in each direction three grids are used for verification and validation. The verification result of the total resistance indicates that the prediction uncertainties are different in three directions, and in each direction the simulation results are validated. In the benchmark model tests, the level of uncertainty of CSSRC's data is quite similar to that of IIHR.

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