

Nonlinear Generated Wave Pattern due to a Three Dimensional Moving Body

Hassan Ghassemi, Mahmoud Ghiasi, Isar Ghamari

Department of Marine Technology, Amirkabir University of Technology, Hafez Ave., 15875-4413, Tehran, Iran, gasemi@aut.ac.ir

Abstract

Steady three-dimensional nonlinear waves generated by surface piercing bodies by the potential based boundary element method (BEM) are presented. By applying Green's theorem and the Green function method, an integral equation for the perturbation velocity potential is obtained based the potential flow theory. Numerical computations are carried out to obtain the free surface waves generated by 3-dimensional surface and submerge moving bodies (like strut, foil and Wigley hull) and some accurate numerical results are presented.

Keywords: Boundary element method, nonlinear free surface, 3D bodies, wave pattern

1. Introduction

When a body moves near the free surface of water, a pattern of trailing gravity waves is generated. Numerical modeling of this problem is a subject of high interest for ship design and marine engineering. An elucidation of such detailed mechanisms is always desirable for research in hydrodynamics. A flow characteristic may be described by two major independent parameters: the Reynolds number and the Froude number. The free surface and wave resistance can be computed with a potential theory, whereas the viscous effect can be considered by using the Navier Stokes equation.

According to linear wave flow theory, a disturbance advancing at steady speed on or below the free surface creates a so-called Kelvin wave pattern. A surface wave created by a disturbance moving at high-speed is considerably different from this theory due to nonlinearity. In such cases, it is necessary to attempt the problem by using a numerical model with nonlinear free surface conditions.

In the past, theoretical free surface and wave resistance was reviewed by Wehausen [1], Newman [2] and numerically presented by many researchers; (Scullen & Tuck [3], Tuck et al. [4], Cao et al. [5], Forbes [6], Parau & Broek [7], Grilli et al. [8]). Very important progress has been achieved in the calculation of two-dimensional free surface waves. Forbes [6] presented a numerical algorithm for the solution of 3-D problems involving a fluid with a free surface. Recently generalisation for two submerged disturbances were also presented by Parau et al. [8]. Nakos & Sclavounos [9] pointed out such a steady flow not only yields wave resistance but also sinkage and trim, which are two significant parameters for determining ship hull power requirements and operating condition. Recently, wave pattern and wave resistance of surface piercing bodies by a boundary element method and also time domain three-dimensional fully nonlinear computations of steady body-wave interaction problem on Wigley and ship hulls were predicted [10][11].

This paper concentrates on the computation of the free surface for a body in steady motion, by means of a potential model for the fluid and a nonlinearized free surface boundary condition. The governing equations are the Laplace equation with slip boundary conditions on the domain. The

free surface boundary condition amounts to a Neumann boundary condition with a source term proportional to the streamlined second derivative of the potential. The three-dimensional problem for moving bodies is considered and the results are presented.

2. Mathematical formulation

We consider a three dimensional moving bodies at a constant velocity U at the surface of a fluid of infinite depth. The sketch of the body and flow are given in Fig. 1. The fluid is assumed to be inviscid and incompressible and the flow to be irrotational. We choose a Cartesian frame of reference moving bodies and assume that the flow is steady. The Cartesian coordinates (x, y, z) is chosen with the z -axis directed vertically upwards and the x -axis in the opposite direction of the velocity U . The equation of the free surface is denoted by $z = \zeta(x, y)$. The potential function is introduced by $\Phi(x, y)$ so its velocity is given by Φ_x and Φ_y in the x and y direction, respectively. In the flow field, the potential function $\Phi(x, y, z)$ satisfies Laplace equation.

$$\nabla^2 \Phi = 0, \quad x, y \in R, \quad z < \zeta(x, y) \quad (1)$$

Three boundary conditions on the free surface are given as follows:

i. Kinematic boundary conditions:

$$\Phi_x \zeta_x + \Phi_y \zeta_y = \Phi_z, \quad z = \zeta(x, y) \quad (2)$$

ii. Dynamic boundary condition:

$$\frac{1}{2}(\Phi_x^2 + \Phi_y^2 + \Phi_z^2) + g\zeta + \frac{p}{\rho} = \frac{U^2}{2}, \quad z = \zeta(x, y) \quad (3)$$

Here, g is the acceleration of gravity, ρ is the fluid density and p the prescribed distribution of pressure.

iii. Radiation condition:

$$\text{no wave as } x \rightarrow -\infty \quad (4)$$

Green's function for a free surface is given by

$$G = \frac{1}{4\pi} \frac{1}{r} \quad (5)$$

Where

$$r = |q - p| \quad (6)$$

r is distance between the field point $p(x^*, y^*, z^*)$ and singular point $q(x, y, z)$.

Applying the integral representation for the potential function $\phi(p)$, Green's second identity can be written as

$$e\phi(p) = \iint_S \left[\phi \frac{\partial G}{\partial n} - G \frac{\partial \phi}{\partial n} \right] ds \quad (7)$$

e is the solid angle, the value of which depends on the position of the field point p in the fluid domain. If point p is placed on the boundary (body surface), then the coefficient e is replaced by

0.5. If point p is placed inside and outside of the body then the value of e is one and zero, respectively. We now choose $\phi = \Phi - \phi_{in}$ and assume that S consists of the free surface and a body surface in the region $z < \zeta(x, y)$. Substituting parameters in the Eq. (7), we obtain

$$\frac{1}{2}(\phi(p) - \vec{U} \cdot \vec{X}(p)) = \frac{1}{4\pi} \int_S (\phi(q) - \vec{U} \cdot \vec{X}(q)) \frac{\partial}{\partial n_q} \left(\frac{1}{r} \right) dS + \frac{1}{4\pi} \int_S \frac{U \zeta_x}{r} dS \quad (8)$$

where $r = \sqrt{(x - x^*)^2 + (y - y^*)^2 + (z - z^*)^2}$.

We introduce dimensionless variables by using U as the unit velocity and L as the unit length. Combining equations (2) and (3) and using the chain rule of calculus we obtain:

$$\frac{1}{2} \frac{(1 + \zeta_x^2) \phi_y^2 + (1 + \zeta_y^2) \phi_x^2 - 2\zeta_x \zeta_y \phi_x \phi_y}{1 + \zeta_x^2 + \zeta_y^2} + \frac{\zeta}{F_n^2} + \frac{P}{\rho U^2} = \frac{1}{2} \quad (9)$$

Where $F_n = U / \sqrt{gL}$.

3. The numerical scheme

The wetted body surface and free surface are discretized to the quadrilateral elements. The discretized form of integral Eq. (3) for the wetted surface of the body and free surface can be expressed as

$$\phi(P_i) = \sum_{\substack{j=1 \\ (i \neq j)}}^{N_B} \phi(Q_j) [DB_{ij}] - \sum_{j=1}^{N_B} \left(\frac{\partial \phi(Q)}{\partial n} \right)_j [SB_{ij}] - \sum_{j=1}^{N_F} \left(\frac{\partial \phi(Q)}{\partial n} \right)_j [SF_{ij}] \quad P_i \in (S_B \cup S_F) \quad (10)$$

where

$$DB_{ij} = \frac{1}{4\pi E} \int_{S_B} \frac{\partial G_{ij}}{\partial n} dS_j, \quad SB_{ij} = \frac{1}{4\pi E} \int_{S_B} G_{ij} dS_j, \quad SF_{ij} = \frac{1}{4\pi E} \int_{S_F} \frac{1}{r_{ij}} dS_j, \quad (11)$$

and N_B and N_F are the number of elements on the wetted hull and free surfaces, respectively. The velocity component $(\partial \phi / \partial n)$ is known on the body surface from kinematic boundary condition, while that is unknown on the free surface. In order to satisfy the free surface boundary condition, $\phi(P_i \in S_F)$, $e = 1$, (from equation 10) is substituted into Eq. (7). Then, we have

$$\sum_{j=1}^{N_B} \phi_j \frac{\partial^2 [DB_{ij}]}{\partial x^2} - \sum_{j=1}^{N_B} (-\vec{V}_S \cdot \vec{n} + v_{BL})_j \frac{\partial^2 [SB_{ij}]}{\partial x^2} + \sum_{j=1}^{N_F} \sigma_j \left(\frac{\partial^2 [SF_{ij}]}{\partial x^2} - K_0 \delta_{ij} \right) = 0, \quad i = 1, 2, \dots, N_F \quad (12)$$

For $\phi(P_i \in S_B)$, $E = 0.5$, Eq. (30) is expressed as

$$\sum_{j=1}^{N_B} \phi_j [\delta_{ij} - DB_{ij}] - \sum_{j=1}^{N_B} (-\vec{V}_S \cdot \vec{n} + v_{BL})_j [SB_{ij}] + \sum_{j=1}^{N_F} \sigma_j [SF_{ij}] = 0, \quad i = 1, 2, \dots, N_B \quad (13)$$

where δ_{ij} is Kronecker delta function. The total numbers of unknowns are $N_B + N_F (= N_T)$. N_B is the number of potential (ϕ) on the wetted body surface and N_F is the number of velocity components (σ) on the free surface. The matrix form of combined equations (12) and (13) are expressed as

$$\begin{bmatrix} [\delta - DB]_{N_B \times N_B} & [SF]_{N_B \times N_F} \\ [DB_{xx}]_{N_F \times N_B} & [-K_0 \delta + SF_{xx}]_{N_F \times N_F} \end{bmatrix} \begin{Bmatrix} \{\phi\}_{N_B \times 1} \\ \{\sigma\}_{N_F \times 1} \end{Bmatrix} = \begin{bmatrix} [SB]_{N_B \times N_B} & [0]_{N_B \times N_F} \\ [SB_{xx}]_{N_F \times N_B} & [0]_{N_F \times N_F} \end{bmatrix} \begin{Bmatrix} \{-\vec{V}_S \cdot \vec{n} + v_{BL}\}_{N_B \times 1} \\ \{0\}_{N_F \times 1} \end{Bmatrix} \quad (14)$$

Here, the second derivative of the influence coefficients (DB_{xx} , SB_{xx} , SF_{xx}) are computed by four-point finite difference operator and also four-point upstream operator is introduced to satisfy the condition of no waves propagating upstream.

Another matrix form of equation (14) is

$$[A]_{N_T \times N_T} \{x\}_{N_T \times 1} = \{b\}_{N_T \times 1}, \quad (15)$$

For this type of problem, a formal solution may be given by the direct solution methods of *LU* decomposition or Gaussian elimination. However, the solution vector may have extensively large components whose algebraic elimination, when multiplied by the matrix A , may give a poor approximation for the right-hand vector b . This affects the errors in the solution of the matrix Eq. (15). In the present study, a SVD technique has been adopted to solve matrix Eq. (14).

4. Numerical results

We applied the BEM with nonlinear free surface condition to the different bodies and present the free surface profile solutions. We found that the results are qualitatively similar. The wake and the two different family of waves (transverse waves and short-length divergent waves) can be easily observed. Figures 2 and 3 illustrate the wave pattern of the strut with *NACA0012* section at two Froude numbers 0.3 and 0.4. When F_n increases the amplitude of the divergent waves becomes more important than that of the transverse waves. The wavelength of the transverse waves increases with the Froude number.

Wave pattern is also shown in Figure 4 when foil (aspect ratio= $A.R.=1.0$) is pierced at immersed ratio ($H/C=0.4$) and $F_n=0.3$. The algorithm can be easily modified to include two bodies and to study the interaction of the wakes produced by each of them. We present an example in Figure 5 for two moving parallel bodies at $F_n=0.40$. The V-shape of the waves downstream becomes in that case a W-shape. This case can be viewed as the wave interactions between ships moving parallelly in deep water.

The method is extended to the standard Wigley hull. Its geometric surface is mathematically defined as

$$y = \pm \frac{B}{2} \left[1 - \left(\frac{2x}{L} \right)^2 \right] \left[1 - \left(\frac{z}{T} \right)^2 \right] \quad (16)$$

where L is the hull length, B the full hull beam and T the hull draft. For the standard Wigley hull used in this computation, the length-to-beam ratio L/B is 10 and the beam-to-draft ratio B/T is 1.6. The computed wave patterns for the standard Wigley hull at Froude number 0.26 and 0.3 are shown in Figures 6 and 7. These are three dimensional views of the free surface. There are no wave reflections at the side- and downstream-boundaries since they are open boundaries for the computational domain. Figure 8 presents the comparisons of the wave elevations along the Wigley hull between the experimental measurements and the numerical results from the current nonlinear solutions for Froude numbers 0.250, 0.267, 0.289 and 0.316, respectively. The nonlinear numerical values agree well with the experimental measurements over the series of Froude numbers considered. The nonlinear calculations indicate significant approximation to the actual free surface elevations along the hull and the wave profiles are quite smooth.

Figure 9 shows comparison of pressure coefficient along Wigley with experimental data the same Froude numbers with wave elevation. The numerical results for non-linear free surface also agree well with experimental data both free surface elevations and pressure distributions for the Wigley hull.

5. Conclusion

We have calculated three dimensional generated waves by 3D moving bodies. These bodies are strut, foil and Wigley hull. The method has been applied to a variety of computational cases and compared with experimental measurements. It is shown that satisfactory agreement and accuracy can be achieved. The three-dimensional wave patterns present a clear and apparent view of the characteristics of free surface wave.

References

1. Wehausen JV. The wave resistance of ships. *Advances in Applied Mechanics*, 1973, Vol. 13, pp93–245.
2. Newman JN. *Marine Hydrodynamics*. MIT Press: Cambridge, MA, U.S.A., 1977.
3. Scullen D. C. and Tuck E. O.: Three-dimensional steady state nonlinear free-surface flow computation, *Gazette of the Australian Mathematical Society* 23 (1996) 80-84.
4. Tuck, E.O. and Scullen, D.C., “A comparison of linear and nonlinear computations of waves made by slender submerged bodies”, *Journal of Engineering Mathematics*, 2002.
5. Cao, Y., Schultz, W. W., Beck, R. F.: Three dimensional desingularized boundary integral methods for potential problems, *Int. J. for Numer. Meth. Fluids* 12 (1991) 785-803.
6. Forbes L. K.: An Algorithm for 3-Dimensional Free-Surface Problems in Hydrodynamics, *Journal of Computational Physics* 82 (1989) 330-347.
7. Parau, E., Vanden-Broeck, J.-M. : Nonlinear 2D and 3D-dimensional free surface flows due to moving disturbances, *European Journal of Mechanics B/Fluids* 21 (2002) 643–656.
8. Grilli, S. T., Guyenne P. and Dias, F.: A fully nonlinear model for three-dimensional overturning waves over arbitrary bottom. *Int. J. for Numer. Meth. Fluids* 35 (2001) 829-867.
9. D. Nakos, P. Sclavounos: Ship motions by a three-dimensional Rankine panel method, *Proc. 18th Symp. Naval Hydro.*, Ann Arbor, August 1990, (National Academy Press, Washington D. C., 1991) 21-40.
10. Sakir Bal: Prediction of Wave Pattern And Wave Resistance of Surface Piercing Bodies ay A Boundary Element Method, *International Journal For Numerical Methods In Fluids*, Vol. 2007.
11. Fuat Kara, F., Tang, C.Q., Vassalos, D., : Time domain three-dimensional fully nonlinear computations of steady body–wave interaction problem, *Ocean Engineering* 34 (2007) 776–789.

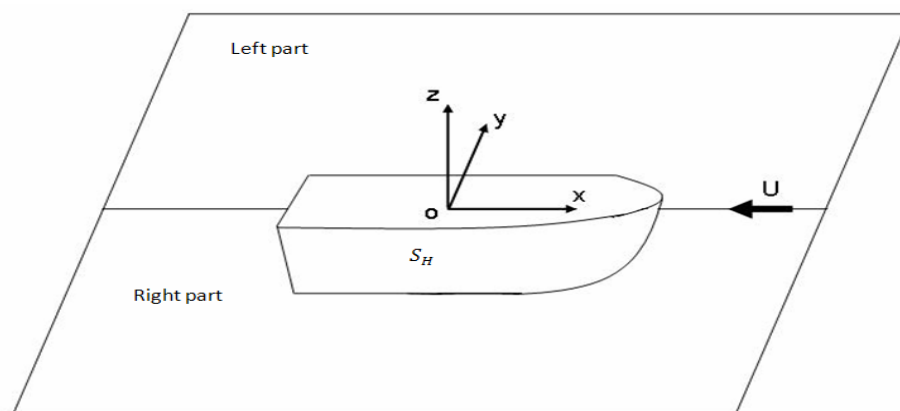


Figure 1: Definition of coordinate system

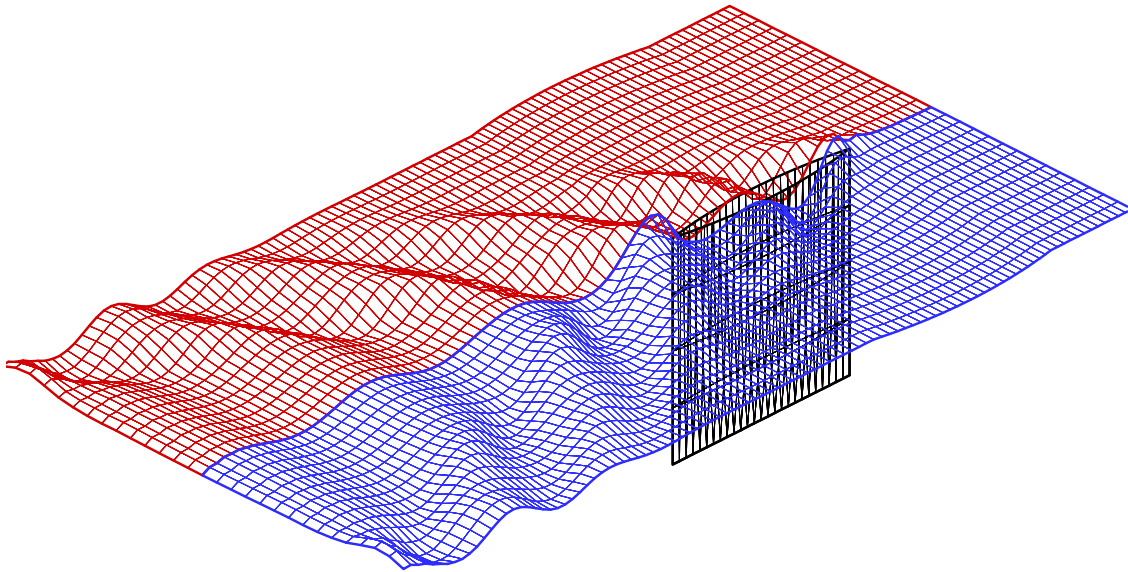


Figure 2: Free surface elevation on the strut with section of NACA0012, ($Fn=0.3$)

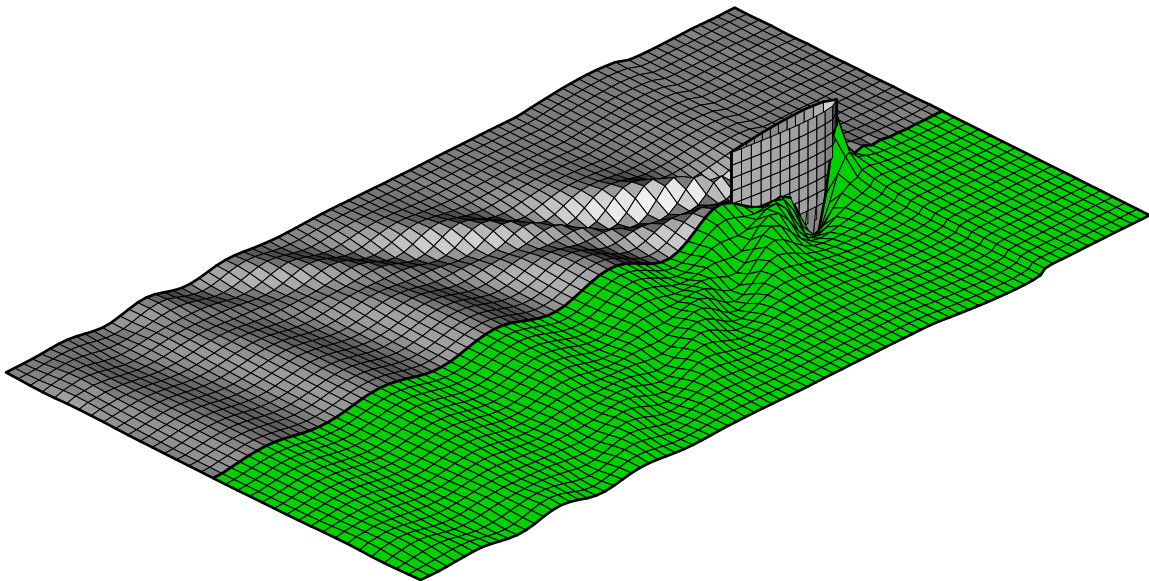


Figure 3: Free surface elevation on the strut with section of NACA0012, ($Fn=0.4$)

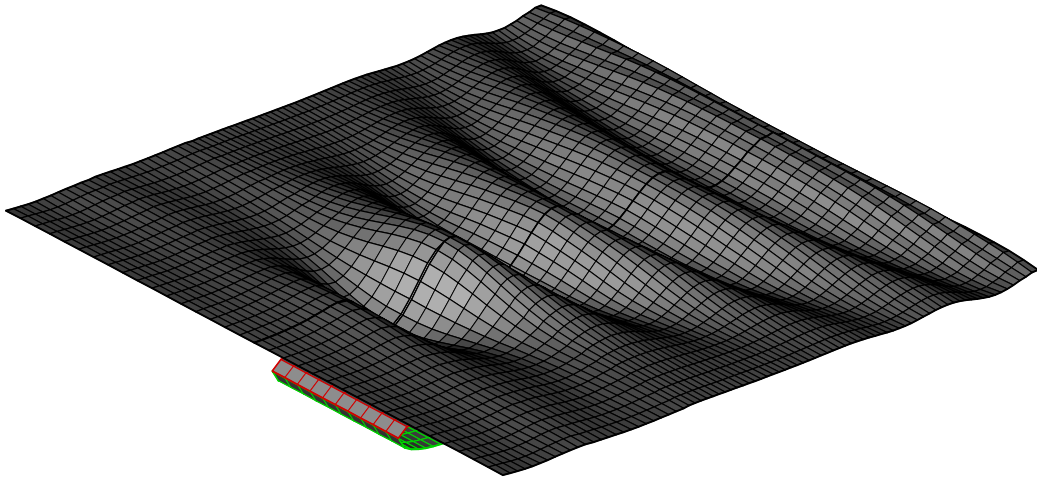


Figure 4: Wave pattern on the Foil ($H/C=0.4$, $A.R.=1.0$, $Fn=0.3$)

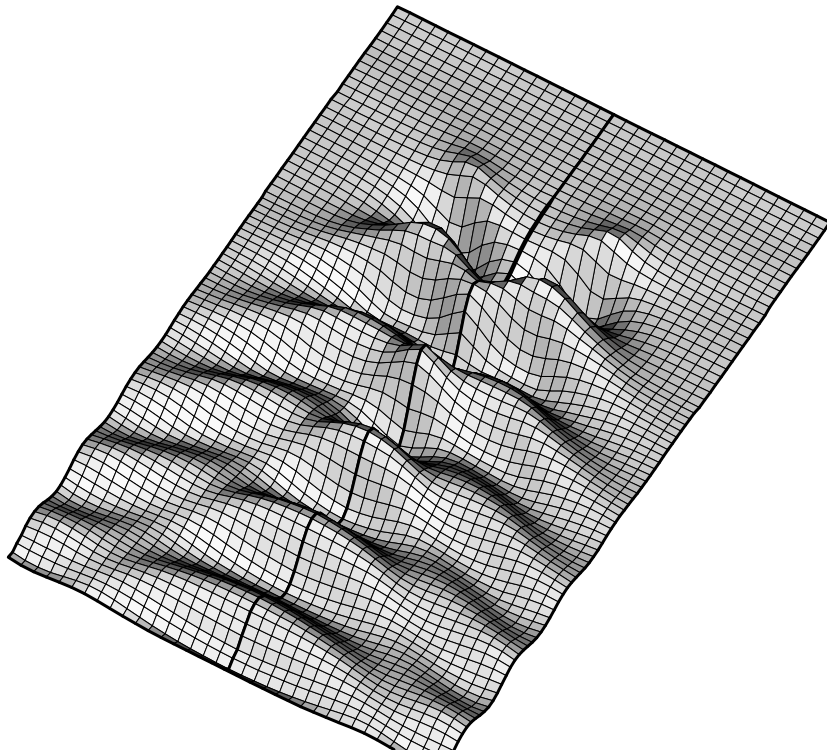


Figure 5: Free surface pattern on the two moving parallel bodies ($Fn=0.40$)

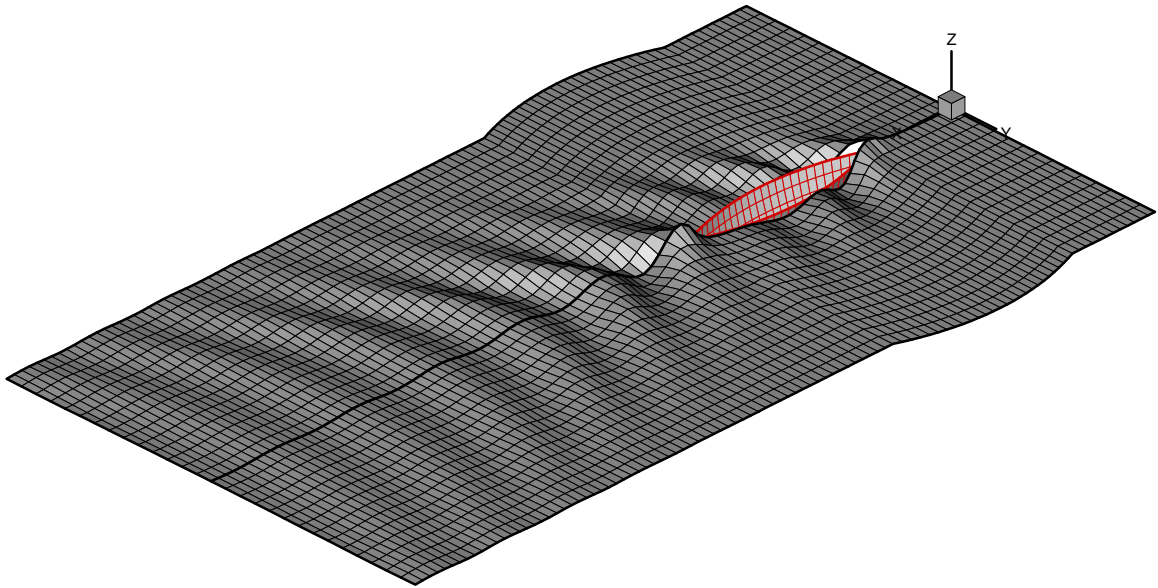


Figure 6: Free surface elevation on the Wigley hull, ($Fn=0.26$)

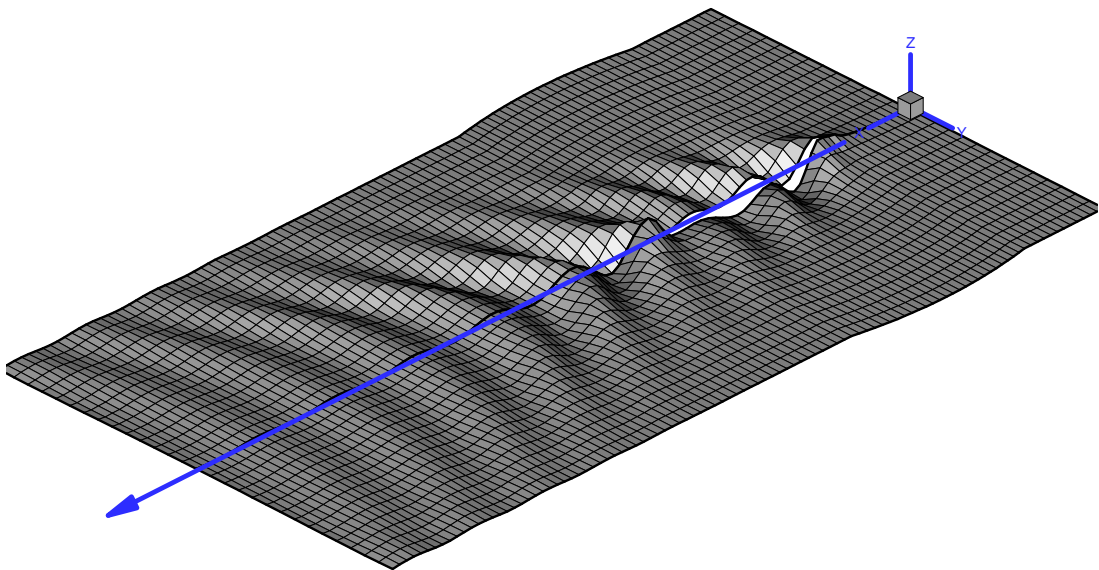


Figure 7: Free surface elevation on the Wigley hull, ($Fn=0.30$)

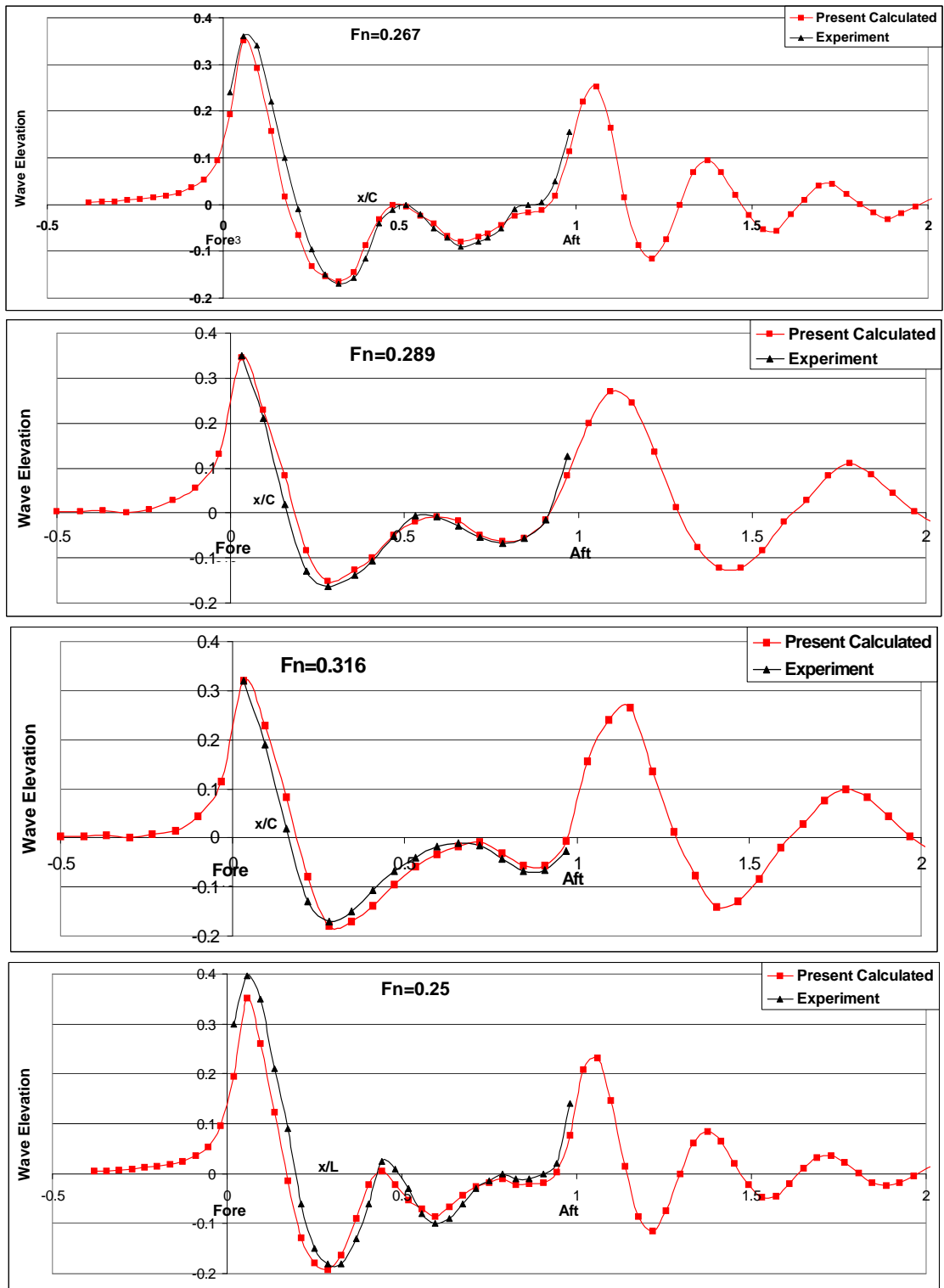


Figure 8: Comparisons of wave elevation along Wigley hull at various Froude numbers (F_n)

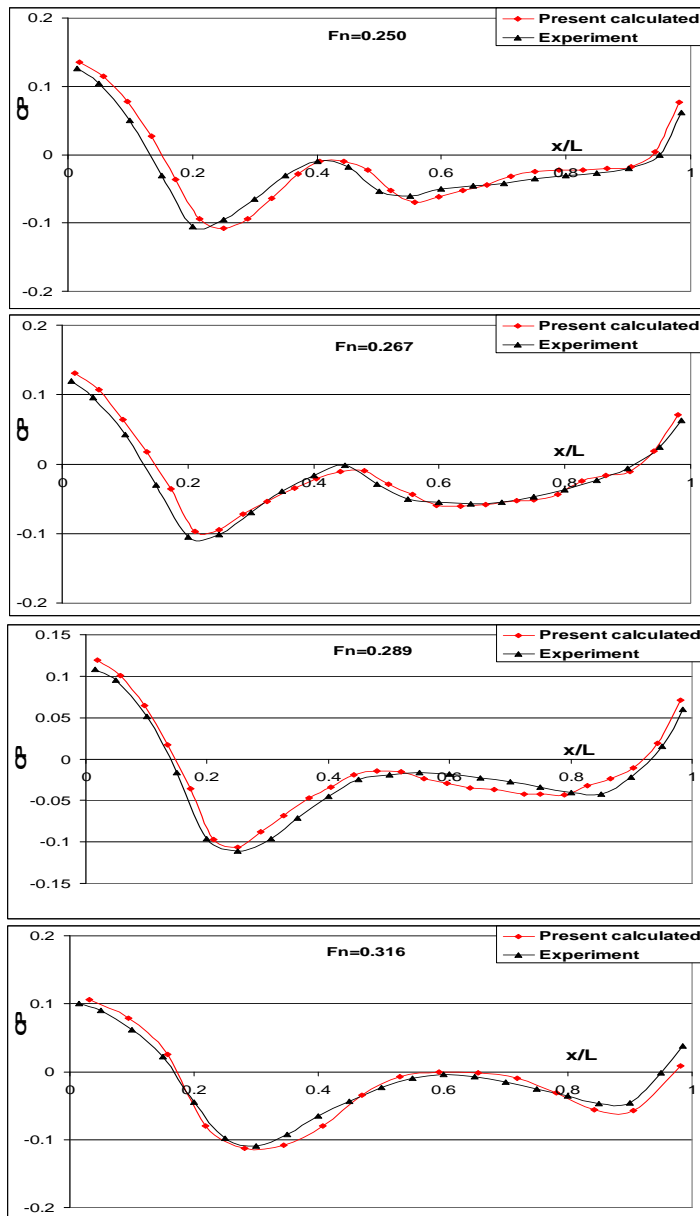


Figure 9: Comparisons of pressure coefficient along Wigley at various Froude numbers (Fn)