

# Simulation of water wave – coastal structure interactions by Smoothed Particle Hydrodynamics

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**ABSTRACT:** The present study deals with an investigation of the flow around a horizontal submerged plate in an open wave flume. This marine structure can be seen as a breakwater near shores. Indeed, when submitted to a regular wave train, the plate partially reflects incident waves leading to a complex flow upstream, downstream and on the top of the obstacle. The wave damping is due to interactions between the plate, the free surface and the seabed. To better understand these phenomena, we have run some simulations of this problem. The Smoothed Particle Hydrodynamics (SPH) method has been chosen thanks to its well known ability to deal with wave propagation, free-surface flows and complex wave breaking problems. Numerical results of the presented SPH model are compared to analytical and experimental results.

## 1 INTRODUCTION

For coastal protection, a submerged horizontal plate can be an economical and sufficient solution to reduce the transmitted wave energy. Some experiments show that, for this kind of use, the plate should be positioned in the vicinity of the free surface to reach the best reflection rate. Such a marine structure is also used for oyster-farming in shallow water. In this case, the plate is fixed close to the seabed.

There are many studies on the estimation of incident and reflected waves by a submerged plate. Siew & Hurley (1977) and Patarapanich (1984) have proposed some analytical models to predict the behaviour of the plate. These models are often based on a linear approximation of the problem, under the shallow water assumption. More accurate solutions have been proposed by Le Thi Minh (1989), who took into account discontinuities by introducing evanescent modes. Some zero and maximum reflection conditions have been deduced from this experimental and analytical data, but the whole reflection mechanism is not well known. Thus, numerical simulations can be helpful to better understand these processes. Patarapanich & Cheong (1989) have performed simulations by means of a Finite Element Method (FEM) of which results are in good agreement with experimental data in terms of reflection rate. Boulier (1996) saw that vortices were periodically emitted at the edges of the plate. He used a Vortex Particle method to study their propagation.

Most of the numerical methods (Boundary Element Method, Volume of Fluid) experience some difficulties for the account of all the fluid dynamics and processes involved in this kind of flow: free surface flow with breaking, shear flow close to edge of the plate, vortices and possibly interactions between the fluid phase and the seabed. Here, the Smoothed Particle Hydrodynamics (SPH) method is used thanks to its ability to easily model free surface flows with strong interface motion. Since it is a lagrangian particle method, it is well suited to simulate several phases with fewer difficulties than with other methods. Indeed, no interface tracking algorithm is needed to see the interface evolution.

In this paper, we propose a study of the flow around a submerged horizontal plate with SPH. First, the Euler equations are written in the SPH framework. Our model is validated on the well-

known case of a dam break followed by an impact on a vertical wall, showing a good agreement with previous results from Colagrossi et al. (2003). The model is then applied to wave propagation with or without a submerged horizontal plate in an open wave flume. Some technical details concerning wave generation, boundary conditions and wave damping are given. Finally, our results are compared to experimental and analytical data.

## 2 SMOOTHED PARTICLE HYDRODYNAMICS

The Smoothed Particle Hydrodynamics (SPH) method has been developed for simulations of gravitational systems in Astrophysics simultaneously by Gingold & Monaghan (1977) and Lucy (1977). SPH is a lagrangian mesh free particle method. It is efficient for computational fluid dynamics in opened domains. The system is modelled by a set of particle which carry their own properties (momentum, pressure) and move according to chosen conservation laws. A numerical approximation of the system solution is then obtained. Monaghan (1994) proposed its extension to free surface incompressible flows by the so-called weakly compressible approach, which is explained below in section 2.2. Since, SPH has been widely applied to a large variety of non-linear flows: interfacial flows, incompressible viscous flows, flows through porous media, impacts simulations and explosions. Among all the free surface flows studies with SPH, one can cite the work by Colagrossi et al. (2003), who presented an improved model for simulating interfacial involving impacts. Dalrymple et al. (2006) also applied SPH to wave overtopping of a horizontal plate.

Although SPH is well adapted to model violent flows, its extension to slow dynamics problem is not straightforward. Energy dissipation can occur when propagating long waves with small amplitudes. Following the work from Vila (1999) and Oger et al. (2007) the standard SPH method can be improved to reduce dissipation and phase-shifting.

### 2.1 Integral representation of a function

The SPH formalism is based on the integral representation of a function. So, to compute a field value at a position  $\mathbf{x}_i$ , the following interpolation formula is used:

$$\langle f(\mathbf{x}_i) \rangle = \int_{\Omega} f(\mathbf{x}) W(\mathbf{x}_i - \mathbf{x}, h) d\mathbf{x} \quad (1)$$

where  $\Omega$  is the computational domain and  $W(\mathbf{x}_i - \mathbf{x}, h)$  is a smoothing kernel. Here a normalized two-dimensional Gaussian function with compact support of radius  $\delta h$  is used:

$$W(s, h) = \frac{e^{-\left(\frac{s}{h}\right)^2} - e^{-\left(\frac{\delta}{h}\right)^2}}{2\pi \int_0^{\delta} s \left( e^{-\left(\frac{s}{h}\right)^2} - e^{-\left(\frac{\delta}{h}\right)^2} \right) ds} \quad (2)$$

with  $s = \|\mathbf{x}_i - \mathbf{x}\|$ . In the simulation presented below,  $\delta = 3$  and  $h = 1.33\Delta x$  have been used,  $\Delta x$  defining the initial inter-particle spacing.

The integral representation can also be applied to compute the first derivative of a function by:

$$\langle \nabla f(\mathbf{x}_i) \rangle = \int_{\Omega} f(\mathbf{x}) \nabla_{\mathbf{x}} W(\mathbf{x}_i - \mathbf{x}, h) d\mathbf{x} \quad (3)$$

## 2.2 Application to the incompressible Euler equations

The Euler equations (4) for mass and momentum conservation of an incompressible fluid, expressed in the lagrangian form, are:

$$\left\{ \begin{array}{l} \frac{D\mathbf{u}_i}{Dt} = -\frac{(\nabla p)_i}{\rho_i} + \mathbf{g} \\ \nabla \cdot \mathbf{u}_i = 0 \\ \frac{D\mathbf{x}_i}{Dt} = \mathbf{u}_i \end{array} \right. \quad (4)$$

where  $\mathbf{u}_i$ ,  $p_i$  and  $\rho_i$  are respectively the velocity, the pressure and the density of the  $i^{\text{th}}$  particle with location  $\mathbf{x}_i$ .

Here, to treat the incompressibility of the fluid, the so-called ‘‘weakly compressible’’ approach is used. The fluid is considered compressible, the pressure varying as a function of density variation according to an equation of state:

$$p_i = \frac{\rho_0 c_s^2}{\gamma} \left( \left( \frac{\rho_i}{\rho_0} \right)^\gamma - 1 \right) \quad (5)$$

where  $c_s$  is the sound speed in the modelled fluid;  $\rho_0$  is the density of fluid at the rest and  $\gamma$  is the polytropic constant ( $\rho_0 = 1000 \text{ kg.m}^{-3}$  and  $\gamma = 7$  for water). Since the fluid is considered compressible, the continuity equation is solved:

$$\frac{D\rho_i}{Dt} = -\rho_i (\nabla \cdot \mathbf{u}_i) \quad (6)$$

The numerical speed of sound is defined to restrict the maximum density oscillations to 1%, according to the following relation:

$$\frac{u_{\max}^2}{c_s^2} \approx \frac{|\delta\rho|}{\rho} \quad (7)$$

In practice, this lead to a sound speed 10 times (or more) larger than the maximum velocity in the fluid  $u_{\max}$ . These problem dependant parameters have a significant impact on the simulation and have to be carefully estimated to obtain an accurate pressure field and thus, accurate results.

Combining equations 1 and 3 with equations 4 and 6, and after some mathematical manipulations, one can obtain the following discretised form of the Euler equations:

$$\left\{ \begin{array}{l} \frac{D\mathbf{u}_i}{Dt} = -\frac{1}{\rho_i} \sum_j (p_j + p_i) \nabla_x W(\mathbf{x}_i - \mathbf{x}_j, h) V_j + \mathbf{g} \\ \frac{D\rho_i}{Dt} = -\rho_i \sum_j (\mathbf{u}_j - \mathbf{u}_i) \nabla_x W(\mathbf{x}_i - \mathbf{x}_j, h) V_j \end{array} \right. \quad (8)$$

$V_j$  being the surface of the  $j^{\text{th}}$  particle.

### 2.3 Implementation

The scheme is stabilized by adding an artificial viscous term in the momentum equation as proposed by Monaghan (1992). The accuracy of the interpolation is also increased by the use of the renormalization technique from Oger et al. (2007).

Equations 8 are time-integrated using a second or fourth order Runge-Kutta scheme, depending on the studied case. At each time step, the largest part of CPU time is spent in constructing the nearest neighbours list, which is achieved by a Tree Search algorithm from Hernquist et al. (1989).

Free-slip conditions on solid boundaries are modelled using the “ghost particle” technique. It consists in mirroring all the fluid particles, whose kernel support intersects the wall. These ghost particles are updated at each time step and their density, pressure and velocity are deduced from the linked physical particle. Other methods (artificial repulsion mechanism described by Monaghan (1994) or arbitrary defined particles of Crespo et al. (2007)) have been proposed to model solid boundaries. The ghost particle method may be more time-consuming than others but, to our opinion, it seems to be the best compromise in obtaining a physical non-penetrating conditions ( $\mathbf{u} \cdot \mathbf{n} = 0$ ). However, its extension to complicated shaped bodies can be difficult.

The free-surface is implicitly enforced by the whole scheme. A specific treatment is achieved to restore the consistency between density, mass and occupied area. The density is periodically re-initialized using:

$$\langle \rho_i \rangle = \sum_j m_j W_j^{MLS}(\mathbf{x}_i) \quad (9)$$

where  $W_j^{MLS}(\mathbf{x}_i)$  is a moving-least-square kernel. It also served to strengthen the pressure condition on the free surface and to smooth the global pressure field.

## 3 APPLICATIONS

### 3.1 Two dimensional dam break

This case has been chosen to validate our model for impact problems and complex interface reconnections. A rectangular box of fluid is kept in rest behind a vertical wall. At the initial time  $t=0$ , the wall is suppressed and the fluid moves under the influence of the gravity. This flow is followed by a violent impact against a vertical wall positioned at the opposite side of the domain.

Figure 1 show the time evolution of the 2D flow generated by a dam break. Our results have been compared to those of Colagrossi et al. (2003) showing a good agreement.

### 3.2 Water waves propagation

Before dealing with the more specific case of a submerged plate in an open wave flume, we wanted to know whether the present formulation could be used to simulate the regular waves propagation. So, this has been achieved by generating a wavelength  $L$  in a rectangular tank of length  $C$  according to:

$$L = \frac{2}{n}C \quad \text{with } n \in \mathbb{IN} \quad (10)$$

Thus, a stationary wave moves into the channel by the superimposition of successive reflected waves from the wavemaker and the opposite wall. Two instantaneous views of particle positions are shown in figure 2 for the  $n = 3$  case. The renormalization technique discussed earlier has been used here to avoid phase-shifting, and the obtained wave train is the desired wavelength. But our model suffers from some dissipation, which could be corrected by coupling the present formulation with Riemann solvers, as shown by Vila (1999).

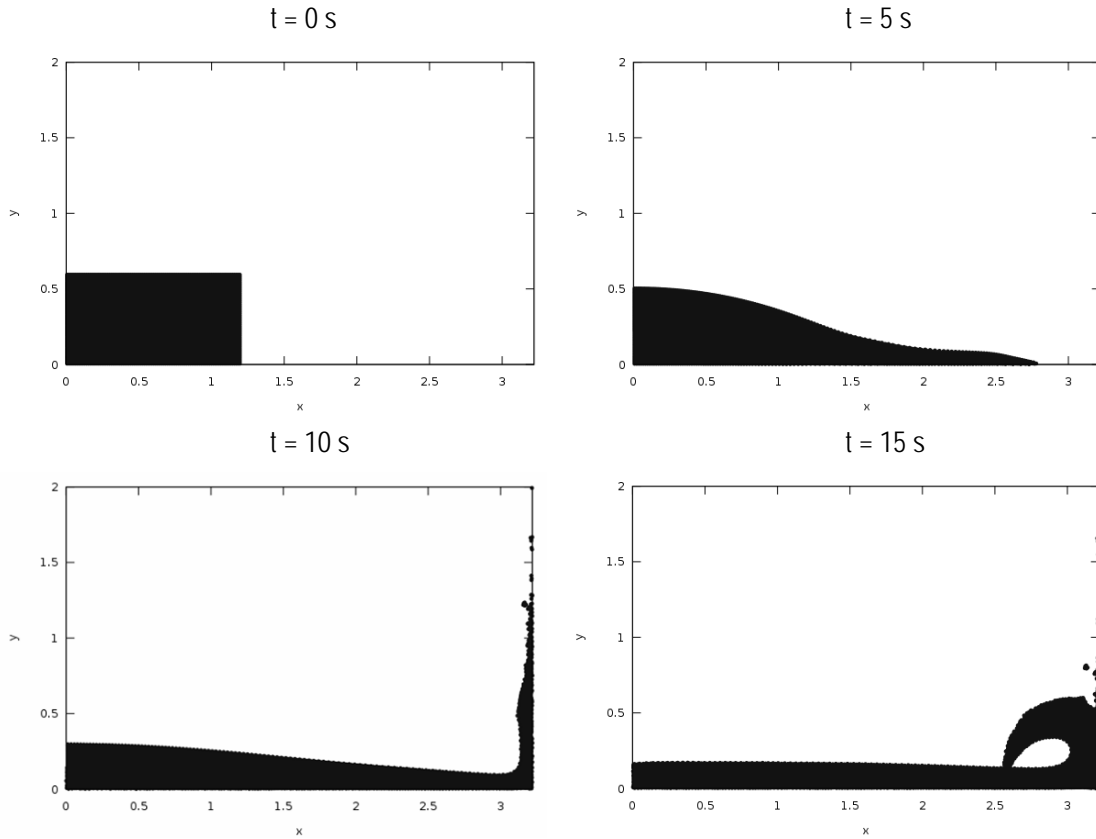


Figure 1. Time evolution of the dam-break flow. The initial fluid volume is 1.2 m length and 0.6 m height. The simulation has been run using 4900 particles

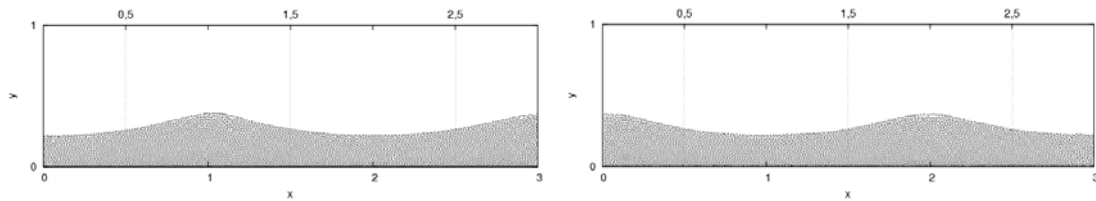


Figure 2. Stationary wave ( $n = 3$ ) in an open wave tank. Particle positions at an instant  $t$  (left) and  $t + T/2$  (right) with  $T$  the wave period. The tank is 3 m length and the wavelength has been set to 2 m.

### 3.3 Wave reflection by a submerged horizontal plate

Previous cases have shown that SPH could model a regular wave train of a desired wavelength propagating in a channel and breaking with interface reconnections. The ghost particles technique has been shown to reproduce a physical full reflection of incident waves on a solid vertical wall. Now, we are interested in reproducing the flow around a submerged plate, as discussed earlier. Notations for this problem are shown on figure 3. Using the same SPH model, waves are generated at given frequencies and are partially reflected from the plate, whose reflection rate depends on the incident wavelength.

A damping zone is introduced at the end of the channel to prevent the transmitted waves to disrupt the whole computation. The first coordinate of acceleration is modified on the last wavelength.

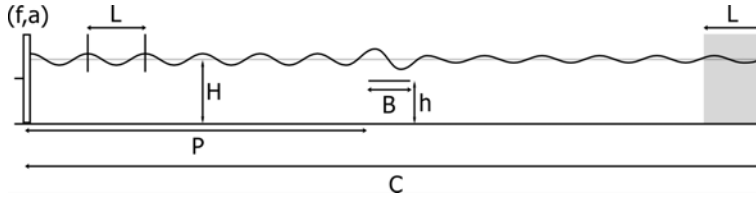


Figure 3. Sketch of the problem of wave reflection by a submerged horizontal plate and adopted nomenclature. Numerical parameters for simulations:  $H=0.2$  m,  $h=0.15$  m,  $B=0.25$  m,  $P=2$  m and  $C=5$  m.

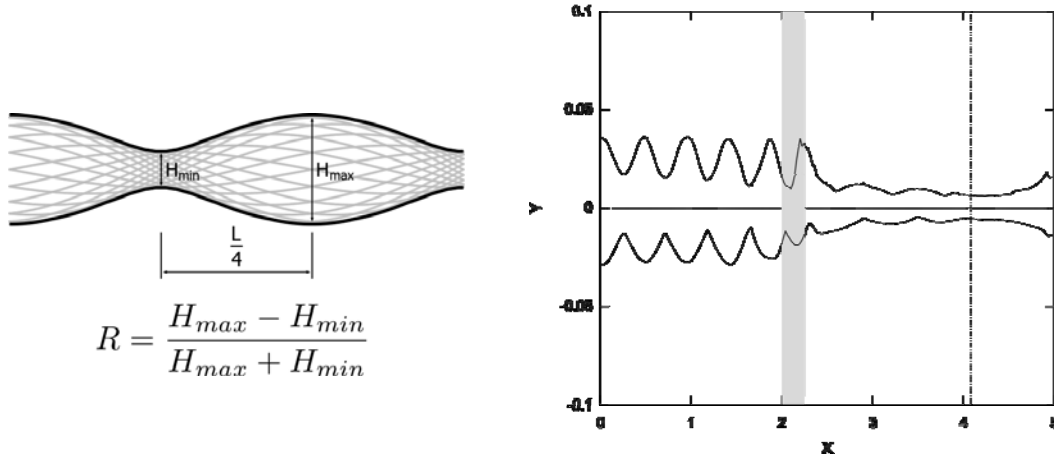


Figure 4. Computation of the reflection rate at nodes and anti-nodes of the maximum and minimum wave heights (left). Maximum free-surface deformation for  $f=1.224$  Hz (right). The grey zone shows the position of the plate.

Both sides of the plate are modelled using ghost particles. A switch in the neighbours search algorithm is introduced to avoid undesired interactions between ghost particles of a given side and fluid particles from the opposite side. This leads to a kind of Kutta-Joukowski condition at the edge of plate.

The time evolution of the free surface is recorded by several numerical probes on the last wavelength upstream the plate. The maximum and minimum wave heights are then used to compute the reflection rate  $R$  of the fundamental mode like it is explained by figure 4. The results are plotted on figure 5. Comparisons between experimental data from Brossard et al. (2008), an analytical solution based on the potential theory from Chagdali (pers. comm.) and our results show that the submerged plate is an efficient breakwater. Its reflection rate depends on its length and on the pulsation of water waves. The maximum is reached when the wavelength over the plate is two times its length. SPH seems to correctly predict the behaviour of the plate if compared to measurements. The theoretical solution tends to overestimate the reflection rate for small wavelengths and be underestimating it for large wavelengths. This should be avoided by adding evanescent modes.

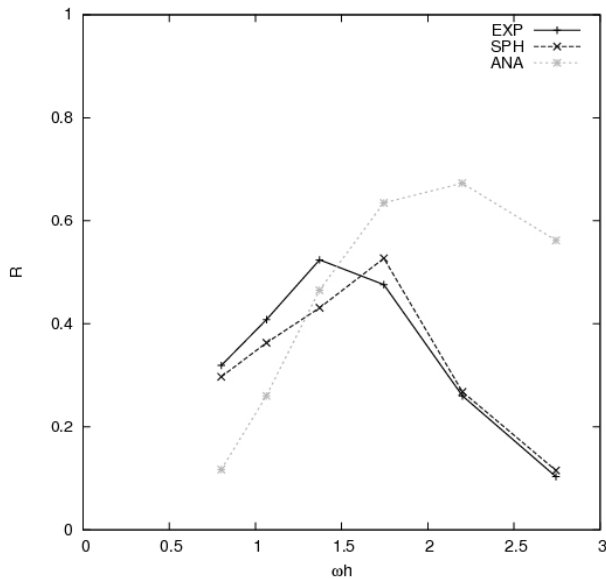


Figure 5. Reflection rate of a submerged horizontal plate as a function of the ratio  $\omega/h$  where  $\omega$  is the incident waves pulsation and  $h$  is vertical position of the plate. Comparison of experimental data from Brossard et al. (2008) (EXP), results of the present method (SPH) and analytical solution from Chagdali (pers. comm.). Corresponding values of wavemaker frequencies are  $f=0.814$  Hz, 1.02 Hz, 1.224 Hz, 1.428 Hz, 1.633 Hz and 1.839 Hz.

#### 4 CONCLUSION

The SPH method can efficiently model fluid-structure interactions as shown with the example of the submerged breakwater. Provided that boundary conditions are accurately imposed, the reflection of simulated waves is in good agreement with some experimental results. Then, simulations should be helpful to investigate all the physical parameters, which have an impact on the plate efficiency.

However, simulations should also help us to better understand the energy dissipation process involved in the wave damping. So, we have to extend our model to Navier-Stokes equations by adding the viscous term in the momentum equation and by applying no-slip conditions on boundaries. Viscous effects will have a significant impact only if the computed pressure field is accurate. As the weakly compressible approach need to be calibrated, we can not ensure that the pressure field is completely correct. Moreover, spurious pressure oscillations appear sometimes near discontinuities. The MLS density re-initialization helps to prevent them but this shows that SPH can give un-realistic results if the scheme is not sufficiently stabilized. This can be done by using Riemann Solvers instead of an artificial viscosity and by carefully defining numerical parameters. But as long as an incompressible flow is simulated using a weakly compressible model, some difficulties might not be avoided.

We are developing a fully incompressible model based on the projection method developed by Cummins et al. (1999). He has shown that the increased stability properties of his formulation allow the use of larger time steps. A pressure Poisson equation need to be solved to enforce the divergence free velocity constraint. But this incompressible approach has not yet been applied to free-surface flows with a satisfactory condition for pressure at the interface. This work is under investigation.

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