

# Turbulent Evolution and Transport in Stably Stratified Flow with a Thermally Sharp Interface

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**ABSTRACT:** In this paper, turbulent evolution and transport properties in stably stratified flow with a sharp interface are studied numerically. Main results are focused on the evolution of mixing layer, turbulent kinetic energy and potential energy, turbulent scale characteristics, transport properties and also the temporal and spatial evolution of scalar field. Main results were analyzed with the effects of stratification and mean shear, it shows that turbulent quantities are depressed by stratification but improved by mean shear, and turbulent counter gradient heat transport, which is found nearby turbulence-generating grid, is closely related to the small-scale motions, which moves in vertical direction.

## 1 INTRODUCTION

Stratified flows exist widely in the ocean, atmospheric boundary layer and some lake and rivers on the continent, it is one of the most important flow in geophysics. Owing to the increasing interest in environmental problems recently, much attention has been paid to the mechanism of turbulent mixing and transport in environmental flows. Turbulent mixing and vertical flux in stratified flows became an important branch in turbulence study, it is of great significance to predict global warming and improve the local climatic model, to solve the problems in oceanic engineering and environment, acoustics and military affairs, and to forecast the diffusion of contaminations especially the scalar diffusion in stratified water flows. Besides the environmental and geophysical problems, practical problems such as the designing of industrial equipment involve stratified phenomena and the elucidation of turbulent mixing in stratified flows is required. Also on the aspect of theoretical study, due to the presence of density gradient in stratified flows, there is effect of buoyancy force except for the effect of inertial force, the process of competition between these two forces dominates the behavior of growth, developing and decay in stratified turbulence.

In the past more than 30 years, many experimental and numerical studies on stratified turbulence have been accomplished to investigate the properties of turbulent mixing, energy transfer, anisotropy, scalar and momentum transport in air and water stratified flows, see the review article of Sherman et al. (1978), Turner (1979), Hopfinger (1987), Fernando (1991), Riley & Lelong (2000), Peltier & Caulfield (2003). Ivey et al. (2008).

It is well known that numerical simulation has been one of the most important methods in turbulence research, so as to stratified turbulence. A number of DNS study on stratified turbulence mainly focused on the large scale structures (Gerz et al.1989, Holt et al. 1992, Jacobitz et al. 1997, Shih et al. 2000), the interaction among local structures (Gerz et al. 1994, Diamessis, & Nomura 2000) and scalar transport property (Holt et al. 1992, Jacobitz et al. 1997, Nagata & Komori 2001). And also there is some research of LES, for instance, Kaltenbach et al. 1994 have investigated the counter gradient transport (CGT) in stably stratified turbulence, and suggested the following research to consider the effect of molecular diffusion on turbulent CGT. But most of the previous studies have paid much attention on the continuously stratified flows,

and until now, there are still some arguments on the application of DNS to the stratified flows with high Pr number in thermal stratification and high Sc number in concentration case for the limited spatial resolution.

So the ideas of this paper, is to investigate the stable thermal stratified flow using LES with the turbulence-generating grid included the computational domain. The characteristics of turbulence velocity fluctuations, the evolution of turbulent mixing layer and turbulent kinetic and potential energy, the evolution of characteristic scales, including the buoyancy scale and overturning scale, turbulent counter gradient transport, and the evolution of temperature field are focused to analyze the effects of shearing and stratification systematically.

## 2 NUMERICAL METHODS

### 2.1 Physical model

Figure 1 shows the physical model of the stably stratified grid-generated turbulence, the computational domain was  $102 \times 10 \times 10$  cm ( $51M \times 5M \times 5M$ ) in the streamwise, vertical and spanwise directions. A round turbulence-generating grid, on which the velocity components are set to zero, was located  $0.02$ m downstream from the entrance. The mesh size  $M$  and the radius of the round rod were  $0.02$ m and  $0.001$ m, respectively. The radius of the round rod we used in the experiment is the same. So we can make quantitative comparison between the measurements and simulations.

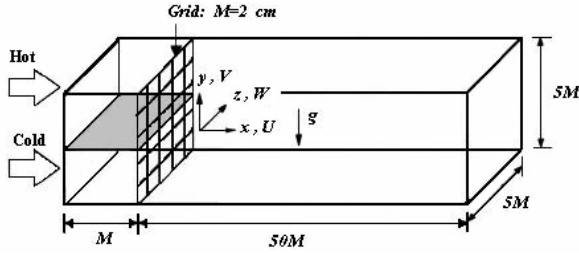


Figure 1. Schematic of stably stratified turbulence.

### 2.2 Equations and model

The governing equations are the incompressible continuity, the momentum equations and energy transport equation, which are spatially filtered (omitted here), and the Smagorinsky-Lilly model is applied to model the subgrid scale stress.

The state equation takes the form

$$\rho = \rho_r(1 - \beta(T_r)T_r + \beta(T)T) \quad (1)$$

where

$$\rho_r = a_0 + \bar{\beta}(T_r)T_r \quad (2)$$

$$\beta(T_r) = \bar{\beta}(T_r) / \rho_r, \beta(T) = \bar{\beta}(T) / \rho_r \quad (3)$$

$$\bar{\beta}(T) = (67939.5 - 9095.3T + 100.2T^2 - 1.1T^3 + 6.5 \times 10^{-3}T^4) \times 10^{-6} \quad (4)$$

and  $a_0$  is constant,  $T_r$  is the environmental temperature of water.

### 2.3 Numerical schemes

The computational domain is  $51M \times 5M \times 5M$ ,  $M = 0.02$ m is the mesh size, and the number of mesh is 1970000. The initial conditions of velocity is induced by a mean velocity profile  $U(z)$  combined with initial turbulent intensity 15%; and the initial temperature distribution  $T(z)$  is determined by different runs, as shown in the table below. And no-reflection condition is

applied at the outlet. The no-slipping and no-penetration conditions were adopted for the upper, lower and side boundaries, which can help us to make quantitative comparison between the measurements and simulations.

The time integration of the N-S equations and transport equations for heat was carried out by implicit scheme with second-order precision, the time step is 0.002s. The spatial terms are approximated by second-order central difference in momentum equation and by second-order windward scheme in temperature state equation.

#### 2.4 Computational runs

To investigate the effects of stratification on turbulent properties, three stratified conditions, namely neutral (Sh-I, II, III), weak (St(1)-I, II, III) and strong (St(2)-I, II, III) stratifications (see table 1), were considered and the initial temperature difference between upper and lower streams corresponded to 0K, 5K, and 10K, respectively. Mean velocities of the upper and lower streams were set to the same value of 0.125m/s.

Table 1. Computational runs.

Runs	$\bar{U}$ (m/s)	$\Delta\bar{U}$ (m/s)	$\Delta\bar{T}$ (K)	$Re_M$
Sh-I	0.125	0.0	0	2500
Sh-II	0.125	0.02	0	2500
Sh-III	0.125	0.05	0	2500
St(1)-I	0.125	0.0	5	2500
St(1)-II	0.125	0.02	5	2500
St(1)-III	0.125	0.05	5	2500
St(2)-I	0.125	0.0	10	2500
St(2)-II	0.125	0.02	10	2500
St(2)-III	0.125	0.05	10	2500

#### 2.5 Validations

To validate the numerical results, we have compared the numerical results of mean temperature to previous work and error function. See the details in Qiu et al. (2008).

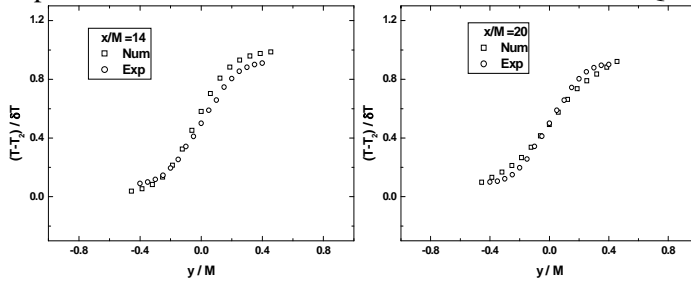


Figure 2. Comparisons of dimensionless temperatures between numerical and experimental results.

Figure 2 shows the comparisons of normalized temperatures between our numerical results and the experimental results of Komori & Nagata (1996) at two sections, i.e.  $x/M=14$  and  $20$ , respectively. It can be inferred from the comparison that there is small difference close to the grid ( $x/M=14$ ), but at the downstream of the flow ( $x/M=20$ ), the numerical results agree well with the experimental results.

### 3 RESULTS AND DISCUSSIONS

#### 3.1 Turbulence intensities

Figure 3 shows the evolution of turbulent intensity of streamwise and vertical fluctuating velocities, normalized by streamwise mean velocity, along the streamwise direction at the centerline of symmetry surface ( $y=0, z=0$ ). In Figure3 (a), (b) and (c), it can be known that the maximum of velocity intensity is increasing with the increasing of mean shear, so it can be inferred that the turbulence intensity is improved by shear effects; and also in Figure3 (a), (d) and (e), we can find that the turbulence fluctuations are suppressed by stratification effects.

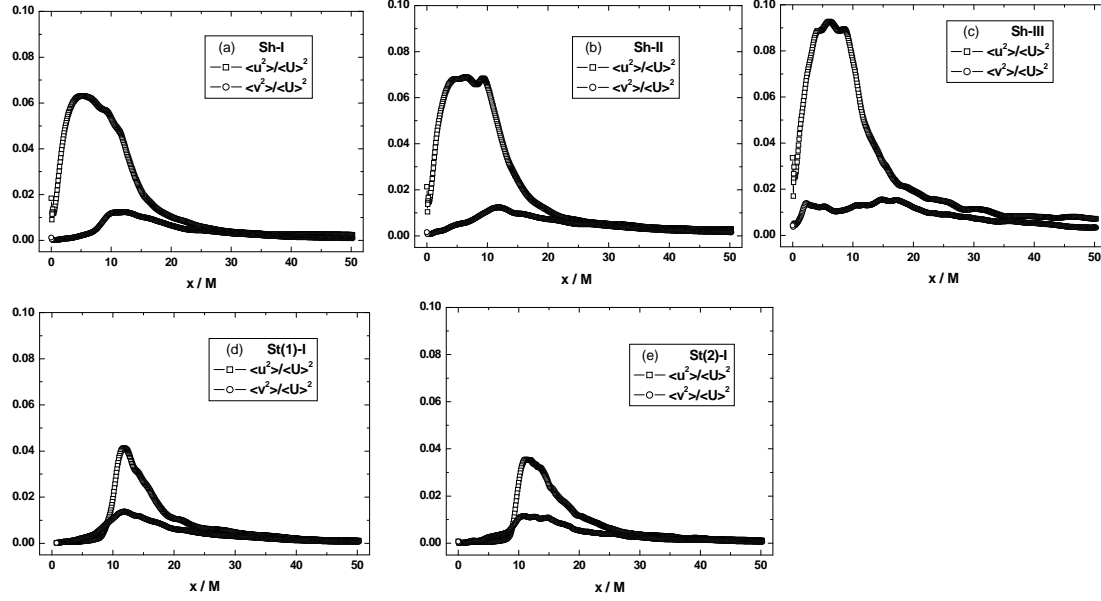


Figure 3. Distributions of turbulence intensity.

In fact, the streamwise evolution of turbulent intensities coincide with the balance relationship between the inertial force and buoyancy force, i.e. when the inertial force is prior to the buoyancy force, turbulence fluctuations is enhanced, otherwise the velocity fluctuations will decay with the development of the flow.

#### 3.2 Evolution of turbulent mixing layer

In this section, the evolution of turbulent mixing layer will be focused to investigate the effects of mean shear and stratification. First, we will define the boundary of turbulent mixing layer by mean temperature.

The thickness of turbulent mixing layer  $l_T$  is defined by  $l_T = (y_2 - y_1)/2$ , where  $T_{y_2}^* = 0.75$ ,  $T_{y_1}^* = 0.25$  are the upper and lower interfaces of the turbulent mixing, respectively and  $T^* = (T - T_{\min}) / (T_{\max} - T_{\min})$ . Let  $T_{y_0}^* = 0.50$ , then we can get the half thickness of the mixing layer,  $l_{T_1} = (y_0 - y_1)/2$  and  $l_{T_2} = (y_2 - y_0)/2$ , which, respectively, represent the thickness of the upper half layer and lower half layer of the mixing layer. Jayesh et.al (1991) and Jayesh & Warhaft (1994) analyzed the evolution of the thickness of turbulent mixing in a wind tunnel, they found that  $l_T = (y_2 - y_1)/2$  increased first, then it became narrower due to the suppression of stratification.

From the evolution of the mixing layer in Figure 4, it can be found that the turbulent mixing is much more improved by mean shear, and also the thickness of upper half layer, which is the side of smaller inlet velocity in shearing cases, is narrower compared to the thickness of lower layer due to the momentum transfer between the two layers. The phenomena that the mixing layer will show an excursion in the shearing cases agrees with the experimental results (Qiu et al. 2008). Considering the stratification effects, we can find that in different stratification cases with same shearing effects, the mixing layer is suppressed by the stratification evidently.

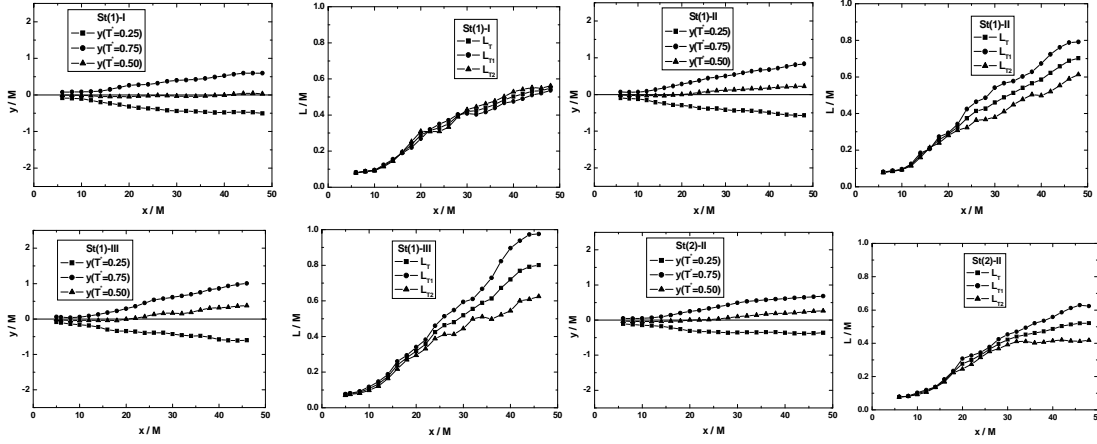


Figure 4. Evolution of turbulent mixing and its thickness

But it should be mentioned here that the mixing layer is defined by mean temperature, which is different with the mixing layer defined by velocity intensity. So we can get different developing figurations of the mixing layer, like the study in the paper of Huq & Britter (1995), who considered the mixing layer through velocity intensity, they show that the mixing layer would grow first when the inertial force is dominating in the developing turbulent flow, then the mixing layer would become narrower gradually because the buoyancy force replaces the inertial force to be the dominating force in the decay turbulence.

### 3.3 Turbulent kinetic energy (TKE) and turbulent potential energy (TPE)

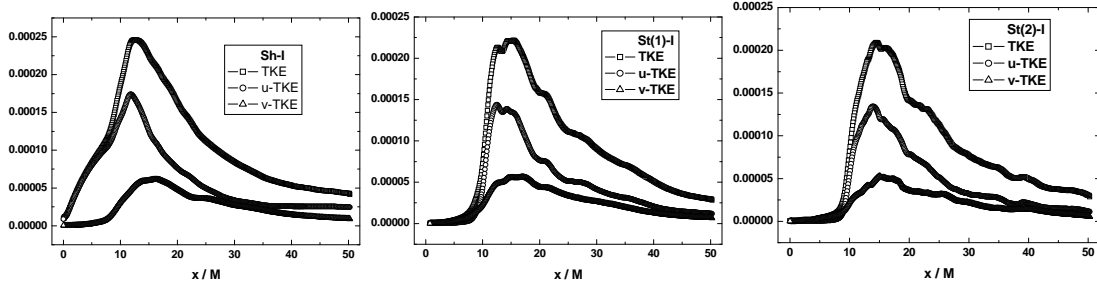


Figure 5 Evolution of turbulent kinetic energy.

Figure 5 shows the longitudinal evolution of turbulence kinetic energy and its components of streamwise and vertical parts. It can be found that in the near-grid region, turbulent kinetic energy enhances due to the effect of mesh grid, and then it decays. In addition, the streamwise component of TKE is distinctly larger than the vertical component of TKE in almost all regions. And in stratified cases, TKE decays much more quickly than in neutral cases, especially in the region dominated by buoyancy force ( $x/M > 20$ ).

The turbulent potential energy TPE is defined as  $TPE = N^2 L_t^2 / 2$ , where parameter  $N = [-(g/\rho_0)(\partial\bar{\rho}/\partial y)]^{1/2}$  is Brunt-Vaisala frequency, which is the frequency at which a vertically displaced particle will oscillate within a statically stable environment,  $L_t = (\overline{\theta^2}/S_t^2)^{1/2}$  is overturning scale or turbulent scale, which is the scale representative of the vertical turbulent excursion of a fluid particle.

Figure 6 shows the longitudinal evolution of turbulent potential energy. We can find that the TPE is increasing along the flow with oscillations. The possible reason is that there is transformation between TKE and TPE, when the fluid particles arrive at the maximum displacement to the equilibrium position, the TPE is in its maximum value, but at this time the vertical TKE is zero. But with the development of the flow, much more vertical TKE is transformed into TPE, so the TPE keep increasing. Also it can be found that when the

stratification is stronger, the TPE increases more rapidly. So it can be inferred that the TPE is much related to the buoyancy force.

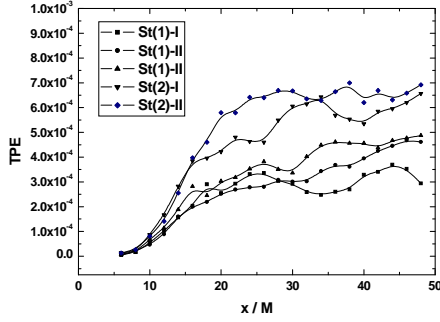


Figure 6. Evolution of turbulent potential energy.

### 3.4 Evolution of length scale

As we know, when a fluid particle moves from the equilibrium position  $y_0$  to another position  $y_0 + \zeta$ , the change of potential energy in per unit volume is

$$\Psi = -\frac{1}{2} g \zeta^2 \frac{d\bar{\rho}}{dy} \Big|_{y_0} = \frac{1}{2} \bar{\rho}(y_0) \zeta^2 N^2 \quad (5)$$

where  $g$  is gravity acceleration,  $N$  is buoyancy frequency mention before. For the fluid particles in same turbulence field, the vertical component of turbulent kinetic energy at position  $z_0$  is

$$\bar{K} = \frac{1}{2} \bar{\rho}(y_0) \bar{v}^2 \quad (6)$$

where  $v$  is vertical fluctuating velocity. Suppose we are considering some certain fluid particles, which moves from position  $y_0$  to another position  $y_0 + \zeta$  and transforms all the kinetic energy into potential energy, then we get  $\Psi = K$ , i.e.,

$$\zeta = \left( \frac{\bar{v}^2}{N^2} \right)^{1/2} = \frac{v'}{N} \quad (7)$$

Here  $\zeta$  can be defined as buoyancy length scale, denoted as  $L_b = \zeta = v'/N$ , which is a measure of the upper limit the particles having ambient kinetic energy can move in the vertical direction, against temperature gradient, where  $v'$  is the r.m.s. of vertical fluctuating velocity.

Similarly, we can define another length scale called overturning scale or turbulent scale, denoted as  $L_t = \theta'/S_t$ , which is the scale representative of the vertical turbulent excursion of a fluid particle. It is a measure of the local potential energy of the flow. Where  $\theta'$  is the r.m.s. of fluctuating temperature,  $S_t = dT/dy$  is the vertical temperature gradient.

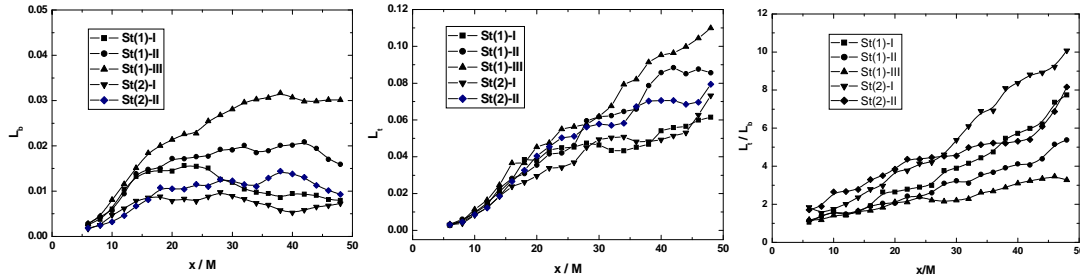


Figure 7. Distributions of buoyancy scale and overturning scale and their ratio.

Figure 7 shows the evolution of buoyancy scale and overturning scale in the streamwise direction. We can find that the buoyancy scale increases first then goes to a certain constant value, stronger the mean shear is, larger the buoyancy scale is; but stronger the stratification is, smaller the buoyancy is; the overturning scale increases also with the developing of the flow, and the mean shear improves the overturning scale.

Referred to the research of Yoon & Warhaft (1990), as  $L_t/L_b$  increases the buoyancy effects will become significant, this result is coincident with the prediction here, because it's more rapid for the overturning scale to rise than buoyancy scale (Fig.7).

### 3.5 Counter gradient heat transport

Gradient transport (GT) and counter gradient transport (CGT) issue are the one of the most important aspect in the study of stratified turbulence. Figure8 shows the distribution of vertical heat flux, which is normalized by the r.m.s. of velocity fluctuations multiplying temperature fluctuations. The vertical heat flux decreases gradually in streamwise direction, the possible reason is that turbulent mixing makes the vertical density gradient smaller, and vertical velocity fluctuations are depressed by stratification effects. It is very interesting that we find a small region, in which the vertical heat flux is positive  $\overline{v\theta} > 0$ . Because the stratified flow is stable, so the mean temperature gradient is positive  $\partial T / \partial y > 0$ , so we have  $\overline{v\theta}(\partial T / \partial y) > 0$ , i.e. the heat transport is counter gradient in this region.

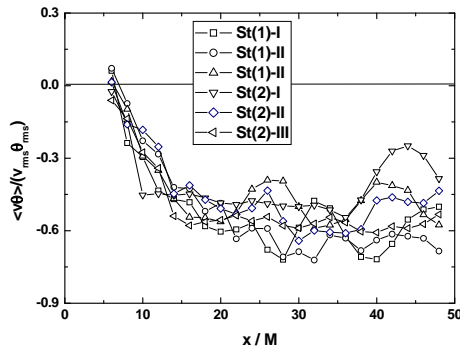


Figure 8. Distribution of vertical heat flux.

### 3.6 Scalar structure

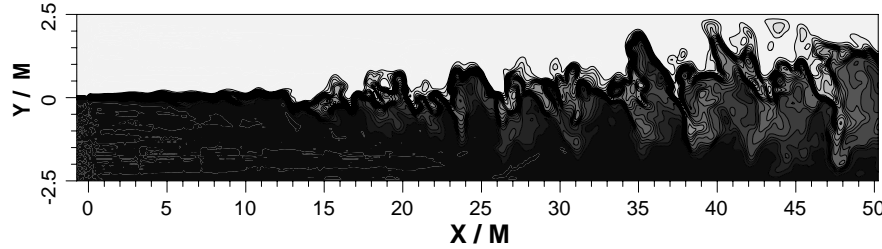


Figure 9. Temperature structure (Run St(2)-III).

Figure 9 shows the evolution of temperature structure in streamwise direction, where the temperature is denoted by the variation of the shade. It can be found that the flow goes through the first instability and second instability during the region of  $10 < x/M < 15$ , then it develops into the mixing layer, we have found this phenomena in the experiment of flow visualization (Qiu et al. 2008). In the mixing process, large scale temperature structures are interlaced, but there is high temperature difference between them. On the contrary, there is little temperature gradient among the small scale temperature motions, it means that turbulent mixing starts from small scale motions, then with the developing of the mixing layer, turbulent mixing prevails to large scale motions. These small scale structures, hanging on the large vertical structures, which formed like fingers (so it can also be called “finger-like” motion), contribute much to turbulent counter gradient heat transport (Komori & Nagata 1996). The counter gradient heat transport phenomena predicted here is basically coincident with previous work of Komori & Nagata (1996), who predicted that CGT appears from  $x/M=6$ .

Through comparing the results of different working conditions of mean shear and stratification, we also find that large scale structures present to be inclined with the effects of

mean shear, and strong stratification depresses the small scale structure and slower the mixing process.

#### 4 CONCLUSIONS

In this paper, turbulent evolution and transport properties in stably stratified flow with a sharp interface are studied numerically. Main results are focused on the evolution of mixing layer, turbulent kinetic energy and potential energy, turbulent scale characteristics, transport properties and also the temporal and spatial evolution of scalar field. Main results were analyzed with the effects of stratification and mean shear, it shows that turbulent quantities are depressed by stratification but improved by mean shear, and turbulent counter gradient heat transport, which is found nearby turbulence-generating grid, is closely related to the small-scale motions, which moves in vertical direction.

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